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Calculation of relative vorticity, and the vorticity area index from NCEP reanalysis data

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Abstract

The calculation of relative vorticity, based on the NCEP reanalysis velocity fields and pressure surface heights, for the period 1958-1999 is discussed. The *vorticity area index* (VAI) is calculated as prescribed in the literature and compared to published values. An imperfect agreement is found, and it is suggested that the difference is due to sensitivity to numerical methods used, as well as to data difference or quality issues.

1 Introduction

The study of atmospheric circulation – in particular the formation of low pressure systems – has been studied using atmospheric vorticity data. An integrated measure of the vorticity in low pressure systems, called the vorticity area index (VAI) has furthermore been constructed and used for studies in the area of solar-terrestrial links (*Roberts and Olson*, 1973).

It is the purpose of this DMI Technical Report to describe the calculation of vorticities, and the VAI index, from the NCEP reanalysis data (Kalnay, *et al.*, 1996), and to compare the values obtained to previously published data. We will also compare to vorticities calculated by others from the NCEP data (Christopher, 2002). Most previous VAI-related work has been based on the geopotential heights in the NMC grid of atmospheric data.

The NMC grid is laid out differently than the NCEP longitude/latitude grid, in that the NMC is laid out as equally spaced points in the stereographic projection plane. This helps the NMC avoid crowding problems near the poles, which is an issue for the NCEP grid. As the NMC and the NCEP grids are to some extent based on the same observations they are not truly independent. We shall seek to explain why the results we get from the NCEP grid is somewhat different from the older NMC-based data, in terms of data differences as well as method differences.

2 Methods

2.1 Calculating vorticities from wind speeds

We calculate the relative vorticity (ζ) from the wind velocity field, by applying the following formula:

$$\zeta = \overrightarrow{k} \cdot (\nabla \times \overrightarrow{V}), \tag{1}$$

which, in polar coordinates (e.g. *Holton*, p.370) can be written as

$$\zeta = \frac{1}{Rcos\phi} \left(\frac{\partial v}{\partial \lambda} - \frac{\partial u}{\partial \phi} \cdot cos\phi + u \cdot sin\phi \right), \tag{2}$$

where (ϕ, λ) are latitude and longitude, R is the radius of the Earth and u, v are the horizontal wind velocity components (from West to East, and from South to North, respectively).

If the velocity data are laid out in a grid in longitude and latitude, the derivatives above can be approximated by central differences:

$$\frac{\partial u}{\partial \phi}(i) \approx \frac{u(i+1) - u(i-1)}{\phi(i+1) - \phi(i-1)} \tag{3}$$

and

$$\frac{\partial v}{\partial \lambda}(j) \approx \frac{v(j+1) - v(j-1)}{\lambda(j+1) - \lambda(j-1)}, \tag{4}$$

where (i, j) are subscripts in the latitude and longitude directions, respectively.

2.2 Calculating vorticities from the geostrophic approximation

An approximation to the relative vorticity can be calculated from pressure gradients, using the geostrophic approximation. The geostrophic vorticity is then, in polar coordinates,

$$\zeta_g = \frac{1}{fR^2 cos^2\phi} \left(\frac{\partial^2 \Phi}{\partial \lambda^2} + cos\phi \left(-sin\phi \frac{\partial \Phi}{\partial \phi} + cos\phi \frac{\partial^2 \Phi}{\partial \phi^2} \right) \right), \tag{5}$$

where Φ is the geo-potential, and $f = 2\Omega sin\phi$ is the Coriolis parameter. The derivatives are again approximated by central differences,

$$\frac{\partial \Phi}{\partial \phi}(i) \approx \frac{\Phi(i+1) - \Phi(i-1)}{\phi(i+1) - \phi(i-1)}, \tag{6}$$

$$\frac{\partial^2 \Phi}{\partial \phi^2}(i) \approx \frac{\Phi(i+1) - 2 \cdot \Phi(i) + \Phi(i-1)}{(\phi(i+1) - \phi(i-1))^2}, \tag{7}$$

$$\frac{\partial^2 \Phi}{\partial \lambda^2}(j) \approx \frac{\Phi(j+1) - 2 \cdot \Phi(j) + \Phi(j-1)}{(\lambda(j+1) - \lambda(j-1))^2}. \tag{8}$$

As a means of internal comparisons and test we also calculate the geostrophic vorticity from the geostrophic winds by application of (2) to u_g, v_g. The geostrophic approximation winds are, in spherical coordinates:

$$u_g = -\frac{1}{fR}\frac{\partial\Phi}{\partial\phi} \tag{9}$$

$$v_g = \frac{1}{fRcos\phi}\frac{\partial\Phi}{\partial\lambda}, \tag{10}$$

and we shall label the variant of the vorticity calculated from geostrophic winds $\zeta_{\overline{g}}$.

For comparison we now calculate and compare the relative vorticities ζ, ζ_g and $\zeta_{\overline{g}}$. We chose level 5 (500 mb) and the 100'th day of 1969. The three fields are shown in figure 4. The fields are evidently somewhat similar, but not identical. The largest range in values is found in ζ_g, with ζ having the smallest range, and $\zeta_{\overline{g}}$ being intermediary.

It is somewhat surprising that the two estimates of vorticity, ζ_g and $\zeta_{\overline{g}}$, that are both based on the geostrophic approximation, are not identical. That ζ, compared to either ζ_g or $\zeta_{\overline{g}}$, does not match is no surprise, as an approximation is involved in calculating the latter two. ζ is compared to maps of the same quantity calculated elsewhere using also the reanalysis velocities (Godfrey, 2002), and an excellent match is found (Figure 5) both for the shapes of the contours and their detailed placement on the map and the values.

Yet another method could be applied to the velocity data in order to calculate the vorticity. The definition of vorticity is the circulation about a closed loop in the horizontal plane divided by the area enclosed:

$$\zeta = \frac{\oint \mathbf{U} \cdot d\mathbf{l}}{A}, \tag{11}$$

in the limit where the enclosed area approaches zero. Letting the area approach zero would be to invite severe numerical problems, but the method could be used to calculate smooth estimates of vorticity by using a suitably-sized area A - the closed loop need not be over the nearest neighbor points. This example serves to remind us that numerical derivatives do not have to be calculated from nearest neighbors, as here, but can be estimated from splines or by using many-point formulae. We seem to have a situation where different difference schemes have given us visibly if not significantly different estimates of vorticity. We have not explored the avenue of finding the optimum numerical scheme for calculating vorticity, however, preferring instead the method based on first derivatives of the velocity field.

The absolute vorticity is the relative vorticity plus f.

2.3 Calculating vorticity area index

The vorticity area index (VAI) is an *ad hoc* measure of the strength of the vorticity, given as the area over which the absolute vorticity is greater than a certain number (*Roberts and Olson*, 1973).

This particular measure of vorticity serves to estimate the size of low pressure systems, and had its origin in work considering the development of low pressure systems in the Gulf of Alaska which appeared to depend on activity related to solar and geomagnetic phenomena.

As the absolute vorticity is dominated by the Coriolis parameter it is largest near high Northern latitudes wherever there is a large contribution from the relative vorticity. In the literature the VAI is generally measured as the area over which the absolute vorticity is greater than $20 \times 10^{-5} s^{-1}$ plus the area over which the absolute vorticity is greater than $24 \times 10^{-5} s^{-1}$.

It is generally stipulated that the VAI should be calculated for a given pressure level and for certain latitudes. Mostly it is calculated for the 500 mb surface and for latitudes above 10 or 20 degrees North. As the absolute vorticity most of the time is below the limits given above it is mainly high Northern latitudes that contribute to the VAI.

We have calculated VAI from ζ for the years 1958-1999 from daily mean NCEP data, north of 10 degrees North. We also calculated values VAI_g from ζ_g and compared them to VAI. We found that they were not identical, although their correlation was 0.82 for the year 1978 and 0.94 for the period 1958-1999. Generally VAI values were smaller than VAI_g values when calculated as above. We ascribe the differences to the errors introduced by the difference schemes and the limitations of the geostrophic approximation. The influence from the chosen difference scheme is not vanishing, however, in that a comparison of ζ_g and ζ_g', calculated using the method in equation 2 but for geostrophic winds, showed that the two relative vorticities were not identical.

Finally, we compared VAI and VAI_g to values published (*Olson et al.* 1977,1979 - the tables for 1946 to 1978 can be down-loaded over the Internet from the NGDC). These published values are derived for the same pressure surface but is calculated twice a day from NMC pressure surface heights data. The NMC grid is laid out in an even grid in the stereographic projections - as opposed to the NCEP grid which is laid out in an even grid in longitude and latitude. The NMC grid is coarser but has the property that numerical problems due to crowding at the poles are avoided. We use NCEP height and velocity fields that are daily means of 4 daily values. We did not find that either of our VAI's reproduced the published values. Their correlation to the '12H' published NMC values were 0.62-0.68, for 1978. The values of VAI_g were closest in range to the published VAI values. We conclude from this comparison and the intercomparison of VAI and VAI_g that the values depend sensitively on the data and the numerical methods used, and recommend that any findings based on VAI be confirmed with the published values and VAI_g.

Further comparison is possible - the VAI data (based on geo-potential heights and NMC data) in *Kirkland et al.* (1996), were compared to the Olson et al. values and those calculated from the NCEP. We find excellent agreement between the Kirkland et al. VAI values and the Olson et al. values for the overlapping years (1959 to 1978). The Kirkland et al. data are based on the extensions of the 500 mb NMC grid available to Olson et al. Comparison to NCEP-derived VAI values is less favorable, as in the case of the Olson et al vs.

NCEP values. While the match between NMC-based values and NCEP-based values (both calculated from geopotential heights) is fair from the mid 1970's to the early 1980's, the NMC VAI values increase by nearly a factor of 2 through the 1980's and into the early 1990's where the Kirkland et al. data we have stop.

Figure 1 shows the two VAI plotted against each other and Figure 2 shows their behavior over time, smoothed to 4-monthly means. The series from 1946-1978 published by Olson et al. is also plotted. Figure 3 shows a segment of the series smoothed only by 1-month running windows. The similarity of the fine structure in the three series is evident, indicating that the differences in the three are not due to outright calculation errors, we think, but rather due to the differences in methods used and the data upon which they are based.

As a further test of the correctness of the routine to calculate VAI from absolute vorticities, we verified that the area of the Earth was recovered when appropriate summation limits were introduced. Thus, we know that the algorithm to calculate VAI from the sum of area elements in which the vorticity exceeds a certain limit is likely to be correct.

2.4 Summary of vorticity calculation and verification efforts

Since it appears that the use of NCEP reanalysis data in place of NMC data gives VAI values that are different it is of the utmost importance for the subsequent analysis that we can verify the correctness of our calculation methods.

Above, we have shown that we are able to use wind velocities to exactly reproduce relative vorticities, by comparing our results to those from an independent calculation. The independent effort used the same data and method but used independently written code. We therefore have confidence in the code we use to calculate relative vorticities, whether from assimilated winds (u,v) or geostrophic winds (u_g, v_g).

Comparing relative vorticities calculated from NCEP geopotential heights and from geostrophic winds calculated from NCEP geopotential heights shows a slight inconsistency (lower two panels of Figure 4), which can mean that the numerical formulae applied are inconsistent. Slight numerical differences are to be expected when difference schemes substitute derivatives, but can we show that the second order difference scheme in equations (5)-(8), substituted for the second derivatives in (5) are inconsistent with application of (3) and (4) to the geostrophic velocities in (9) and (10) where a difference of the form (6) is used?

Replacing the derivatives in (9) and (8) with first differences and applying (3) and (4) to the result and inserting these relationships into (2) shows that second derivatives of Φ will be involve use of data at $j+2, j$, and $j-2$ whereas the second order difference used for the second order derivatives in (5) are evaluated at $j-1, j$, and $j+1$. The numerical derivatives therefore are not the same as the curvature of the surface differenced as far as two locations away from the central point becomes involved in one case and only out to the first neighbors in the other approach, although the analytical formulae are identical, and this leads, we think, to the differences in the relative vorticities seen in the lower two panels of Figure 4. This highlights the importance of the choice of the differencing schemes used in place of derivatives.

As far as the vorticities are concerned we are therefore confident that we are calculating them correctly, although we have a dependency on the numerical differencing schemes used that apparently causes differences in the calculated relative vorticity, near cyclone and anti-cyclone centers of up to 50% or so.

The calculation of VAI from absolute vorticities is next considered. We rely here on a scheme that calculates areas from the sum of area elements that are centered on grid points where the absolute vorticity exceeds the stated limits. Our method passes the simple test of recreating the Earth's surface area if we 'turn on' all elements and sum them once. As we cannot calculate vorticities from the NMC grid we cannot confirm that we can recreate the VAI that are derived from the NMC data, but we can get fairly close when we use NCEP data that is not a daily average but is available at the times also used by the NMC. This is shown in figure 5. As the NMC grid was laid out differently (evenly spaced in the stereographic projection) we have other issues of the dependency on choices in numerical methods to consider, in addition. We have not chosen to perform these verification steps, however, resting instead on having shown that we can calculate VAI from NCEP data that correlates highly with the NMC-based values for both un-smoothed series and smoothed ones.

For identical periods the NMC data and the NCEP data should in principle describe the same state of the atmosphere. However, the methods used in calculating the NCEP reanalysis and the NMC data are not the same, nor are the data except for some overlap.

We therefore conclude by stating that we believe we have applied numerical methods correctly to the NCEP data and have derived VAI values that are close to the earlier published NMC-based values, and that the differences we see are due to differences in choices of numerical methods, the layout of the grids and the subsequent consequences due to choices of numerical methods, and finally, the different relationships between the data in the NCEP and the NMC on the one hand and reality on the other.

Tables of the VAI values derived from NCEP are available from the author.

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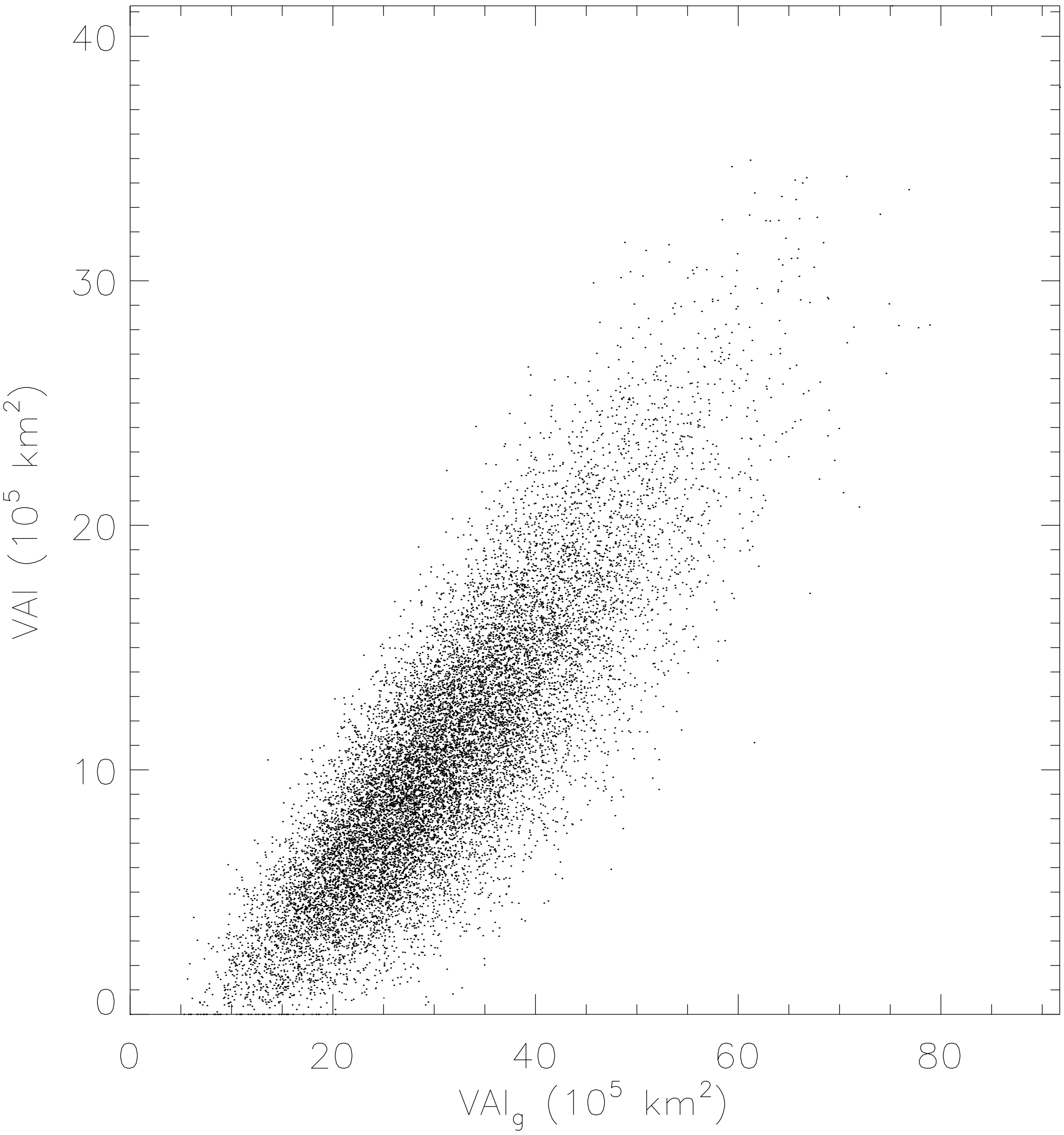

Figure 1: Vorticity areas calculated by two methods compared. VAI$_g$ is calculated from pressure level heights and the geostrophic approximation, and VAI is calculated from the horizontal velocity field. Both at 500 mb for each day from 1958-1999.

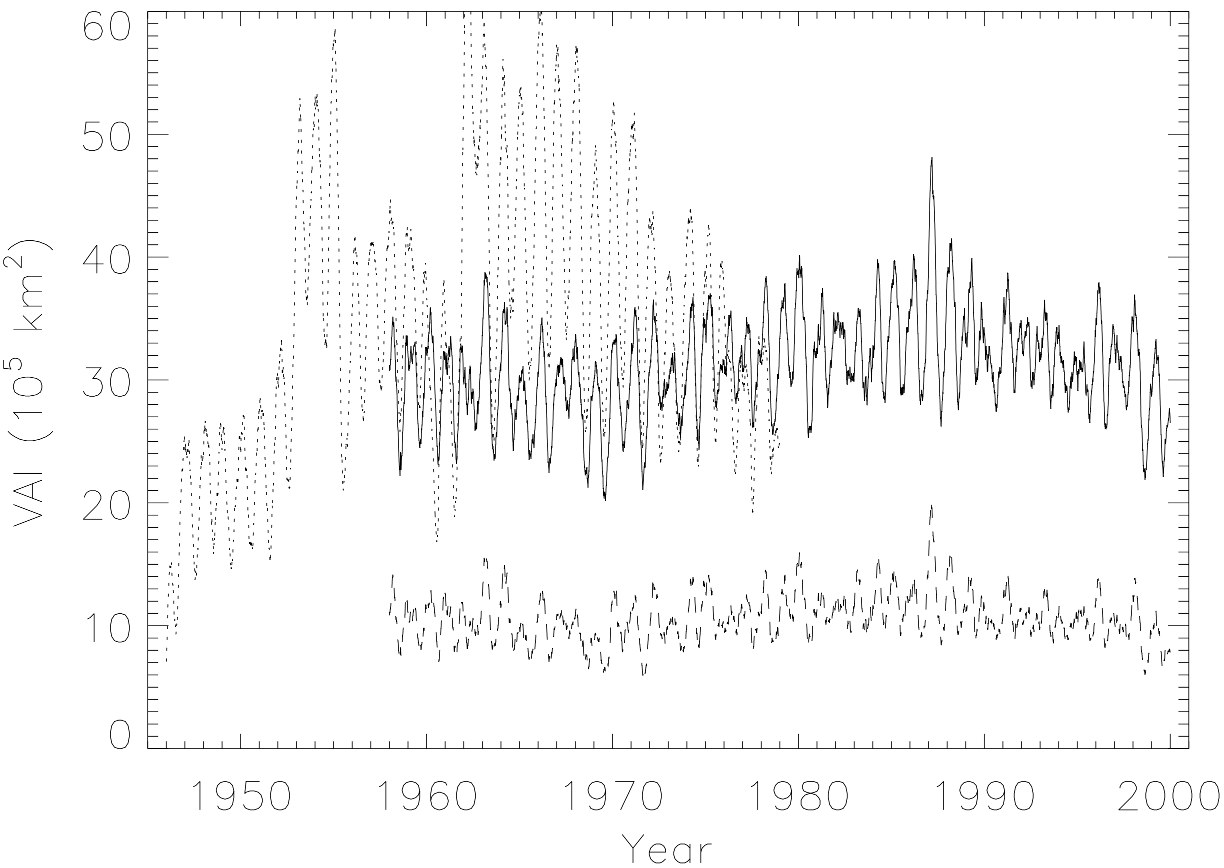

Figure 2: VAI calculated by two methods in this paper, compared to a published set of values. The upper dotted curve is for the VAI published by SCOSTEP (in *Olson et al.*, 1977, 1979). The middle solid curve is for VAI_g (i.e. calculated from pressure level heights at 500 mb), and VAI calculated from the horizontal velocity field is shown by the lower stippled line. For ease of comparison 4-month smoothed values are shown, although daily values are available - for our work based on daily mean NCEP data, and for *Olson et al.* (1977, 1979) the 12UT value is used (two a day are published).

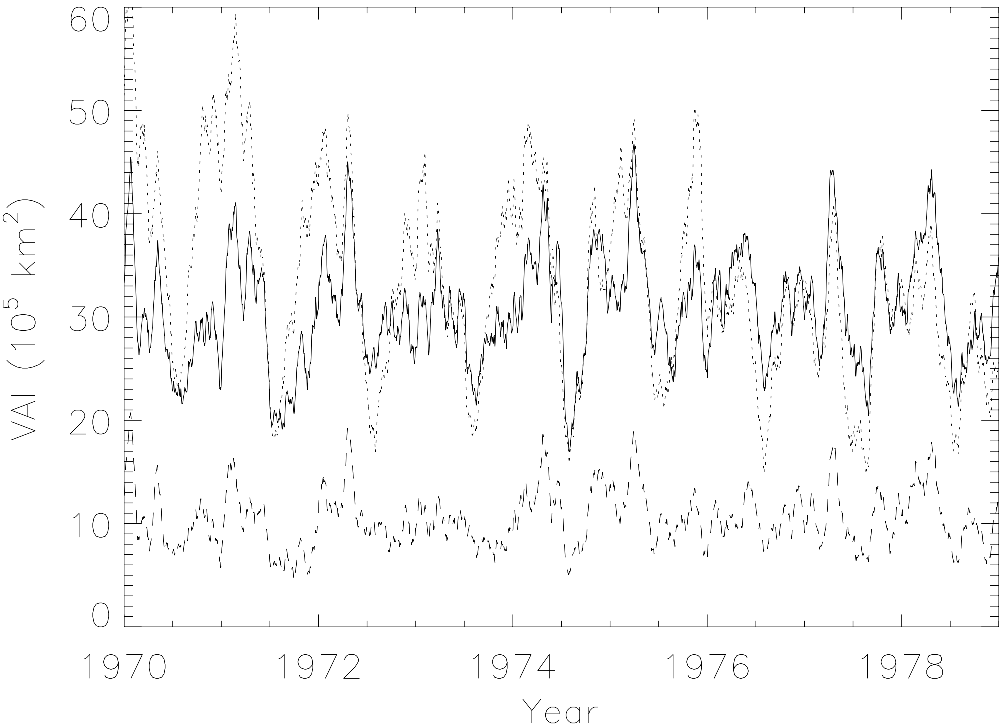

Figure 3: As in figure 2, but for the period 1970 through 1978 and smoothed by one month running windows only. The series share the same annual cycle and most of the fine-structure features are present in all the three series.

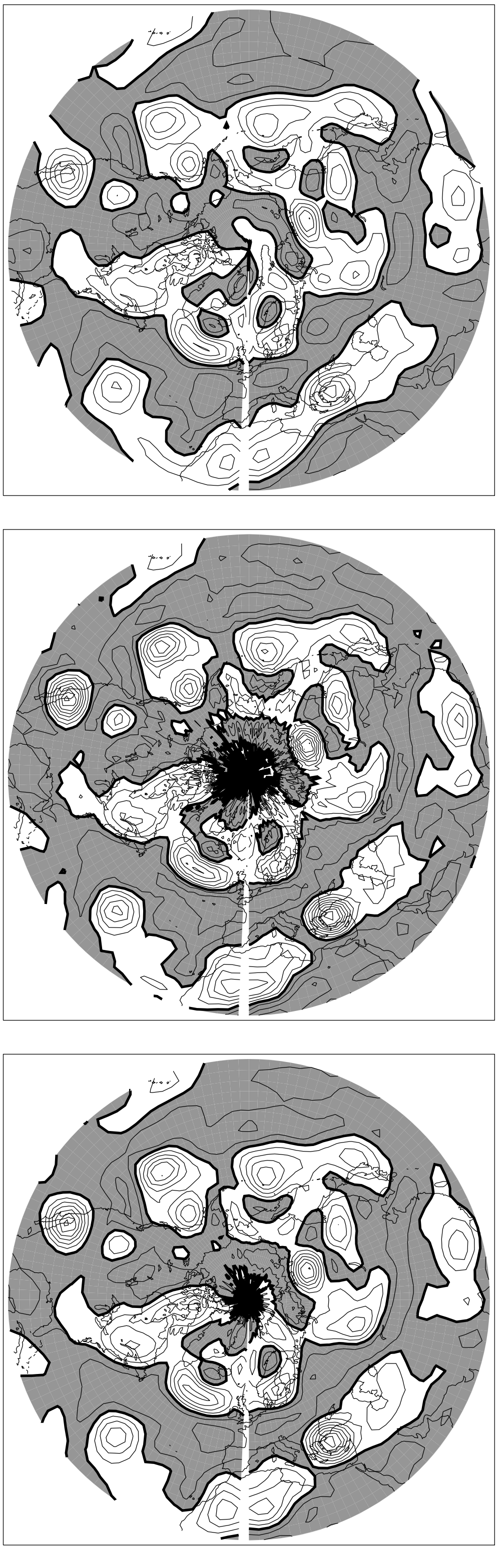

Figure 4: Relative vorticity at the 500 mb pressure surface in the daily-means NCEP reanalysis, in 1969, day 100 (April 9'th) calculated in three different ways. The panels show the relative vorticity north of 20 degrees N. The heavy contour is the zero contour, contour spacing is $2 \times 10^{-5} s^{-1}$, and negative vorticities are shaded. The uppermost panel shows relative vorticity ζ calculated from (u,v), the middle panel shows the relative vorticity ζ_g calculated from geopotential heights, and the final panel shows the relative vorticity $\zeta_{\overline{g}}$ as calculated by first obtaining geostrophic winds from geopotential heights and then calculating the vorticity from these.

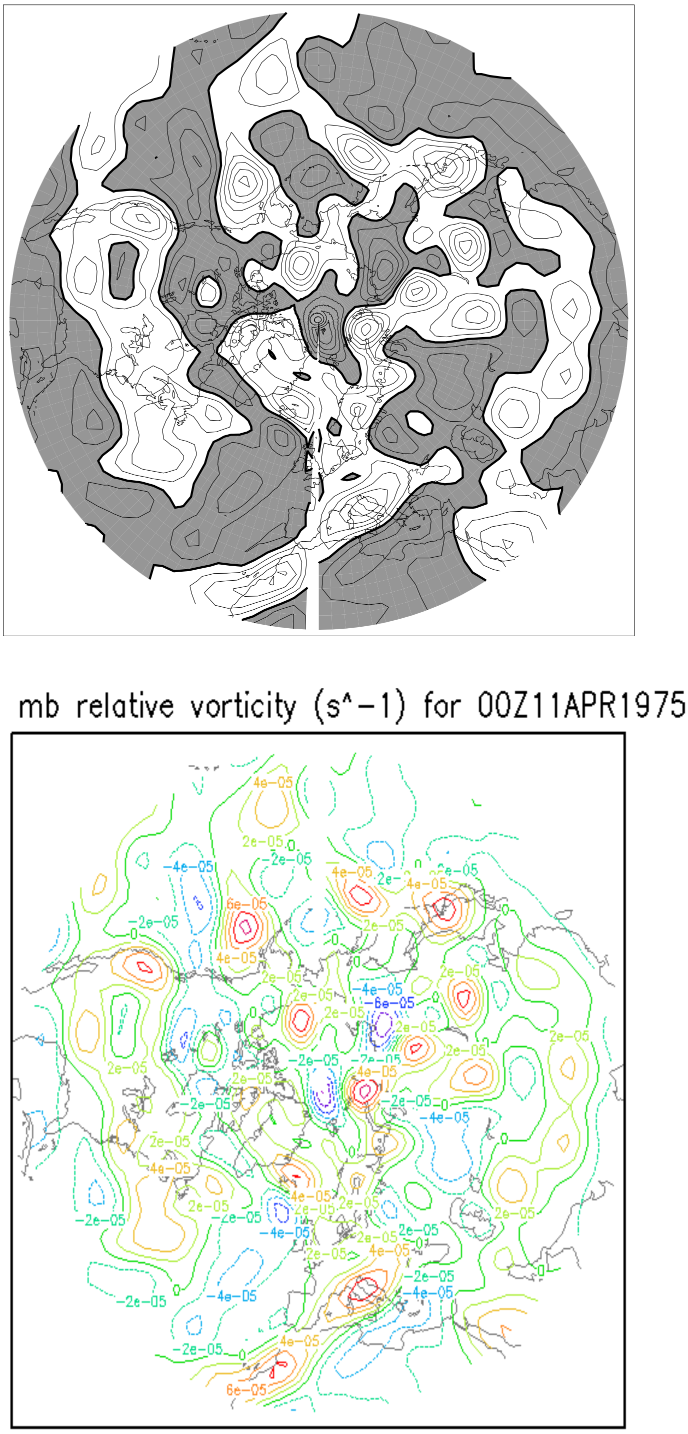

Figure 5: Comparing relative vorticity from 4-times-a-day NCEP data for April 11 at 0UT in level 500mb, for the Northern hemisphere, north of 20 degrees. The upper panel shows the result calculated using equation 2, with equations 3 and 4 in this paper, while the lower panel shows the same calculation performed independently (Godfrey, 2002). In the upper panel the gray areas have negative relative vorticity, the zero contour is heavy and contours are spaced at $2 \times 10^{-5} s^{-1}$, as are the colored contours in the lower panel. Inspection of the contours shows that they are virtually identical.