DANISH METEOROLOGICAL INSTITUTE

——— SCIENTIFIC REPORT ———

01-07

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dmi.dk

COPENHAGEN 2001

ISSN Nr. 0905-3263 (printed)
ISSN Nr. 1399-1949 (online)
ISBN-Nr. 87-7478-444-7

CALCULATION OF THE HEIGHT OF STABLE BOUNDARY LAYERS IN OPERATIONAL MODELS

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Submitted to ***Boundary-Layer Meteorology***

Abstract

Currently used and newly proposed calculation techniques for the height of the stable boundary layer (SBL), including the bulk-Richardson-number method, diagnostic equations for the equilibrium SBL height and a relaxation-type prognostic equation, are discussed from the point of view of their physical grounds and relevance to experimental data. Among diagnostic equations, the best fit to data exhibits an advanced Ekman-layer height model derived recently with due regard to the role of the free-flow stability. Its extension to non-steady regimes provides a prognostic equation recommended for use in operational models.

Keywords: boundary layer height, stable stratification, bulk Richardson number, air-pollution, weather prediction

1. Introduction

The height, h, of turbulent boundary layers (often called "mixing height") is requested in a number of practical applications, first of all, in pollution-dispersion modelling, where the upper boundary of the turbulent layer could play a role of impenetrable lid for pollutants released at the surface. h also appears as a mixing height scale in turbulence schemes within climate and weather prediction models.

Currently used h-calculation techniques are summarised in the final report of the working group 2 "Mixing Height Determination for Dispersion Modelling" of the EU COST Action 710 (Seibert et al., 1998, 2000). This document asserts essential uncertainties in specification of h especially for stable boundary layers (SBLs) and general need for further research with emphasis on non-steady regimes and wave-turbulence interaction.

To some extent these features of the SBLs are already included in the SBL height formulation (Zilitinkevich et al., 2001a). Further analysis along this line is given in the present paper. The state of the art in the SBL height parameterisation can be found in the above quoted papers.

2. Critical Richardson number methods

2.1. Gradient Richardson number

As follows from the classical theory (Taylor, 1931) infinitesimal disturbances in a steady-state homogeneous stably stratified sheared flow decay if the gradient Richardson number Ri exceeds a critical value Ri$_c$,

$$\mathrm{Ri} \equiv \frac{\beta(\partial\theta_v / \partial z)}{(\partial u / \partial z)^2 + (\partial v / \partial z)^2} > \mathrm{Ri}_c = 0.25 . \qquad (1)$$

Here, z is the height, u and v are the velocity components, $\theta_v = \theta + 0.61 T_0 q$ is the virtual potential temperature, θ is potential temperature, q is specific humidity, $\beta = g / T_0$ is the buoyancy parameter, g is the acceleration due to gravity, and T_0 is a reference value of the absolute temperature. The estimate Ri$_c$=0.25 is derived from the perturbation analysis.

Strictly speaking, the above concept it is not immediately applied to turbulent boundary layers, which are always heterogeneous in the vertical and often non-steady. Nevertheless a rather common practice is to employ the critical gradient Richardson number as a convenient tool for distinguishing between the planetary-boundary-layer interior, supposed to be essentially turbulent, and the free atmosphere, supposed to be non-turbulent or only weakly turbulent. Accordingly, the turbulent boundary layer height, h_E , is deduced from inequalities

$$\mathrm{Ri} < \mathrm{Ri}_c \text{ at } z < h_E \text{ and } \mathrm{Ri} > \mathrm{Ri}_c \text{ at } z > h_E , \qquad (2)$$

regardless the type of the boundary layer, stable or unstable.

Leaving aside the general applicability of the above method to heterogeneous flows, it obviously implies that the boundary layer is in the steady state. Hence the critical Richardson number method can provide, at best, the equilibrium height of the boundary layer, h_E , rather than its actual height, h.

In practical calculations the gradients on the r.h.s. of Eq. (1) are approximated by finite differences as $\Delta\theta / \Delta z$, $\Delta u / \Delta z$ and $\Delta v / \Delta z$, where the increments in θ, u and v over the vertical distance Δz are taken from measured or numerically simulated vertical profiles (e.g., Marion et al., 1991).

When the boundary-layer height is known from independent measurements or numerical simulations (e.g., using advanced turbulence closures), the Richardson number immediately above the boundary layer can be identified, and the method of estimating h_E through Eqs (1) and (2) can be evaluated. A reasonable criterion of its robustness would be an empirical evidence that the “geophysical critical Richardson number” is not much variable.

Estimates of Ri_c presented in Table 1 contradict this expectation: Ri_c varies from 0.15 to 7.2. By this means the gradient Richardson number method in the above straightforward form is hardly justified.

Table 1. Geophysical estimates of the critical gradient Richardson number, Ri_c.

Reference	Ri_c	**Comments**
Taylor, 1931	0.25	Theoretical – for homogeneous flows
Webb, 1970	0.19 – 0. 2	Deduced from conventional empirical constants in the Monin-Obukhov theory
Businger et al., 1971	0.21	Deduced from the Kansas experiment based constants in the Monin-Obukhov theory
Businger, 1973	0.15 – 0.5	From wind-tunnel and field data: turbulence is developed at Ri<0.15 and decays at Ri>0.5
Maryon and Best, 1992	1.3 (up to 7.2)	Using calculated Ri(*z*) from numerical model NAME/UM and actual *h* from radiosoundings
Straume et al. 1998	0.55	Best fit for the ETEX experiment data

2.2. Bulk and finite-difference Richardson numbers

An alternative, widely used method of estimating *h* employs, instead of the gradient Richardson number Ri, Eq. (1), the boundary-layer bulk Richardson number, Ri_B, specified as

$$\mathrm{Ri}_B \equiv \frac{\beta \Delta\theta_v h}{U^2} \tag{3}$$

through the wind velocity at the upper boundary of the layer, $U = \sqrt{u^2(h) + v^2(h)}$, and the virtual potential temperature increment across the layer, $\Delta\theta_v = \theta_v(h) - \theta_v(0)$. As common sense suggests, the SBLs can grow on the background of stable stratification only until Ri_B achieves some critical value, Ri_{Bc}. When this threshold is passed, the shear production of turbulent kinetic energy, characterised by the strength of wind *U*, becomes insufficient to overtake the energy losses, characterised by the buoyancy increment $\Delta\theta_v$. This reasoning immediately yields the formula (Mahrt, 1981; Troen and Mahrt, 1986)

$$h_E = \frac{\mathrm{Ri}_{Bc} U^2}{\beta \Delta\theta_v}, \tag{4}$$

where h_E is the equilibrium SBL height.

The fact that any version of the Richardson number method provides the equilibrium rather than actual SBL height deserves emphasising. Generally SBLs, especially over urban or coastal areas, are non-steady. What follows from the above reasoning is only an indication that the actual SBL should have a tendency to evolve towards its equilibrium state with the height given by Eq. (4). It is obvious that the accuracy of the bulk Richardson number method can not be too high. Nevertheless, Eq. (4) gives reasonable order-of-magnitude estimates of h taking Ri_{Bc} in the interval 0.2< Ri_{Bc} <0.5.

In view of rather uncertain specification of Ri_{Bc}, numerous attempts were made to improve the method through a compromise between the gradient and the bulk Richardson number approaches. The idea of this development is to exclude the lower portion of the SBL and to determine a "finite-difference Richardson number", Ri_F, on the basis of increments $\delta\theta_v = \theta_v(h) - \theta_v(z_1)$ and $\delta U = \sqrt{u^2(z) + v^2(z)}\Big|_{z=z_2}^{z=h}$ over the height intervals $z_1 < z < h$ and $z_2 < z < h$. Clearly Ri_F is nothing but a roughly estimated gradient Richardson number. Assuming the existence of its standard critical value, Ri_{Fc}, the equilibrium SBL height formulation becomes

$$h_E \approx \frac{(h_E - z_2)^2}{h_E - z_1} = \frac{\mathrm{Ri}_{Fc}(\delta U)^2}{\beta\delta\theta_v}. \tag{5}$$

There is no consensus in the choice of the lower reference heights. For example, z_1=2 m and z_2=0 in Holtslag et al. (1990), z_1=30 m and z_2=0 in Sørensen et al. (1996), $z_1 = z_2$= 20, 40 or 80 m in Vogelezang and Holtslag (1996). In numerical-weather-prediction (NWP) and other operational models, z_1 and z_2 are usually identified with the lower numerical-model level. Using any of these versions, the finite-difference critical Richardson number Ri_{Fc} remains quite uncertain (see Table 2). Hence, to the authors' understanding, the Ri_{Fc}–method can hardly be considered as an essential step forward compared to the Ri_{Bc}–method.

Besides this principal difficulty, different authors employ different definitions of bulk or finite-difference Richardson numbers. As a result, inattentive readers could easily overlook what particular version of the Richardson number is implied – bulk, Ri_{Bc}, or finite-difference, Ri_{Fc}, and moreover, what particular choice of z_1 and z_2 is implied in the definition of Ri_{Fc}. For reader's convenience, a summary of the Richardson number based methods is briefly presented in Table 2.

Table 2. Alternative critical values of the bulk Richardson number Ri_{Bc} ($z_1 = z_2$=0) and the finite-difference Richardson number Ri_{Fc}.

Reference	z_1, m	z_2, m	$\mathrm{Ri}_{\{B,F\}c}$	Comments
Laikhtman, 1961	0	0	1.65	Ri_B in terms of geostrophic wind, data from Main Geo. Obs. expeditions in Russia
Hanna, 1969	0	0	0.33-0.56	Ri_B in terms of temperature gradients in lower 100 m, data from O'Neill, Nebraska
Melgarejo and Deardorff, 1974	0	0	Average 0.55 typical 0.3	Data from Wangara exp.; h determined through the wind maximum height, h_u
Brost and Wyngaard, 1978	1	1	0.11-0.22	Data from measurements and 2nd order closure model
Anisimova et al., 1978	0	0	up to 7	Lab experiments with down slope drainage flows (analysed by Mahrt 1981)
Zeman, 1979	$\frac{1}{2}h$	$\frac{1}{2}h$	0.5	Data on nocturnal jets over the Great Plains, O'Neill, h compared with the Brost – Wyngaard closure model
Mahrt et al., 1979	2	0	average 0.3-0.5 maximum 15	Data from Wangara, Risø, O'Neill and Haswell; h compared with h_u
Mahrt, 1981	0 2	0	0.5 – 1.0	Typical values of Ri_{Bc} or Ri_{Fc} from different sources
Wentzel, 1983	2	0	0.33	Wangara data (mainly for radiation dominated SBLs) with different estimates of h
Troen and Mahrt, 1986	0	0	0.5	Dara from LES (Deardorff model) and Wangara exp.
Byzova et al., 1989	0	0	0.6-1.0	Data on turbulence and mean profiles from 300-m tower, Obninsk, Russia, 1972-1974
Heineman and Rose, 1990	2	0	0.3-0.55 typical 0.33	Tethered balloon sounding, Filchner/Ronne Ice Shelf, Antarctica; h compared with h_u, the Zilitinkevich (1972) SBL height scale, and the height, h_θ, of the lowest θ gradient discontinuity

Holtslag et al., 1990	2	0	0.25-0.5	Best fit for radiosounding data from de Bilt
Holtslag and Boville, 1993	10	0	0.5	Modelling and radiosonde data from several sites
Sørensen et al., 1996	30	0	0.14-0.24	Ri_B from either HIRLAM or radiosoundings, h from radiosoundings at weakly stable SBLs, Jægersborg
Vogelezang and Holtslag, 1996	20 40 80	20 40 80	(i) 0.21–0.22 (ii) 0.30–0.32	(i) For nocturnal SBLs, (ii) for well-mixed SBLs – both from Cabauw-mast data and SODAR data (for h)
Fay et al., 1997	0	0	0.38	Ri_B from German NWP model and actual h from either radiosoundings or 2nd order closure model
Makshtas et al., 1998	2	0	0.4	Ri_F from aerological and balloon observations over Weddell sea; h compared with wind-maximum and inversion heights (h_u and h_i)
Andreas et al., 2000	0	0	0.4	Ri_B from radiosoundings at the Ice Station Weddell; h compared with h_u and h_i

Besides Ri_B and Ri_F, some other versions of the Richardson number were considered. Thus Vogelezang and Holtslag (1996) proposed for near-neutral SBLs a version of Ri_F modified by including in the velocity increment an additional term dependent on the friction velocity, u_*^2, namely $Ri_F = \beta\delta\theta_v(h - z_1)/[(\delta U)^2 + bu_*^2]$. They took $b \approx 100$ and $z_1 = z_2 = 20$, 40 or 80 m and recommended the critical values of Ri_F equal to 0.16-0.20 for nocturnal SBLs (type I) and 0.25-0.28 for well-mixed SBLs (type II). Using data from the experiment TEBEX, Baltink and Holtslag (1997) considered one more version of this formulation with $z_1 = z_2 \approx 0.1\, h$ and $\mathrm{Ri}_{Fc} = 0.25$.

As clearly seen from the above discussion, any version of the Richardson number method displays the following weak points.

- Inherent uncertainty of the method: it provides an equilibrium rather than actual SBL height and inevitably becomes a poor approximation in non-steady regimes.
- Considerable uncertainty in the choice of appropriate critical Richardson numbers, which additionally degrades the accuracy of the method.

The first disadvantage can not be mastered within the Richardson number method. Instead, a prognostic SBL-height equation should be used to account for non-steady states (see Section 3).

The second disadvantage does not look hopeless. Wide spread in empirical estimates of the bulk or finite-difference Richardson numbers can naturally be treated as an indication that other parameters, besides the wind speed and the buoyancy increments, affect the equilibrium SBL height.

This idea is by no means new. Brutsaert (1972) has found that Ri_{Bc} generally increases in the regimes with clear-air radiation cooling. Arya (1972) reported about pronounced difference in empirical estimates of Ri_{Bc} for the shallow and the deep SBLs. Joffre (1981) disclosed a positive correlation between the critical bulk Richardson number, $\mathrm{Ri}_{Bc} = \beta\Delta\theta h / U^2$, and the dimensionless combination fh/u_*, where f is the Coriolis parameter and u_* is the friction velocity.

2.3. Earth's rotation

Generally the role of the Earth rotation in the Ekman boundary layer is indisputable. However, the above correlation could at least partially be an artefact, as the SBL height h appeared simultaneously on the x- and y-axis of Joffre's diagrams.

Further investigation of the dependence of Ri_{Bc} on f is presented in Figure 1. Here, Ri_{Bc} is plotted against two alternative dimensionless numbers, μ and M, characterising comparative roles of stratification and rotation,

$$\mu = \frac{u_*}{|f|L}, \text{ and } M = \frac{\beta\Delta\theta_v}{|f|U}. \qquad (6)$$

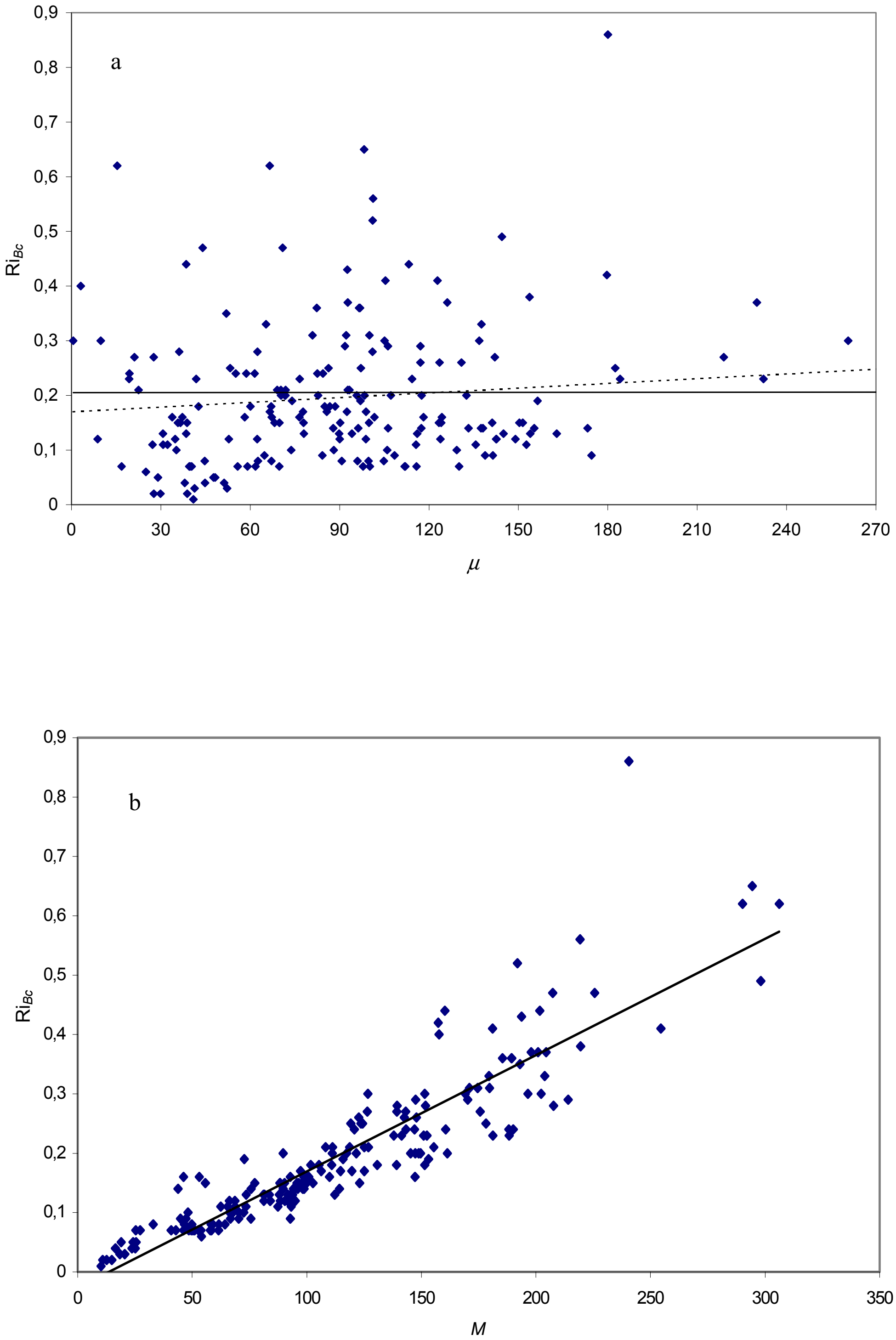


Figure 1. Empirical dependencies of the critical bulk Richardson number $\mathrm{Ri}_{Bc} = \beta\delta\theta h / U^2$ on alternative stratification/rotation parameters $\mu = u_* / |f| L$ and $M = \beta\Delta\theta_v / |f| U$, after Cabauw data: (a) $\mathrm{Ri}_{Bc}(\mu)$ and (b) $\mathrm{Ri}_{Bc}(M)$.

Of these two, M is composed of the bulk increments over the SBL (Section 2.2.1.2 in Zilitinkevich, 1970) and μ, of the surface-layer parameters, namely, the friction velocity u_*, and Monin-Obukhov length scale,

$$L = \frac{-u_*^3}{B_s}, \tag{7}$$

where $B_s = \beta F_{\theta vs}$ and $F_{\theta vs}$ are the near-surface turbulent fluxes of buoyancy and virtual potential temperature, respectively. Experimental data are taken from measurements performed in 1977-1979 on the 200-meter meteorological mast in Cabauw, the Netherlands (Niewstadt, 1984; Van Ulden and Wieringa, 1996; Vogelezang and Holtslag, 1996). They include the mean vertical profiles measured at 8 levels between 2 and 200 meters) and the SODAR-measurement based SBL height*, h*.

Figure 1a does not show pronounced systematic dependence of Ri_{Bc} on μ. This is not surprising bearing in mind that all data are taken from one an the same site, Cabauw, so that the Coriolis parameter is fixed and the representative range of μ is rather limited. A vague tendency of Ri_{Bc} to increase with increasing μ (indicated by dashed line) is not statistically ensured. Further analysis of this type of dependencies including data from measurements at different latitudes (together with data from lab experiments in rotating tanks and large-eddy simulations with variable *f*) would be useful.

A well-pronounced dependence of Ri_{Bc} on M shown in Figure 1b does not say much about Ri_{Bc}, but implicitly indicates that h could depend on f. Indeed, $M = \mathrm{Ri}_{Bc}\, U/|f|h$, which is why Ri_{Bc} factually drops out from linear relationship $\mathrm{Ri}_{Bc} \propto M$. Thus taking $\mathrm{Ri}_{Bc} \approx 0.002M$ (which is a reasonable approximation at $M > 50$) immediately yields $h \approx 0.002U / |f|$.

2.4. Static stability above the SBL

An external parameter, which is not included in the standard Richardson-number methods, is the Brunt-Väisälä frequency, N, in the free atmosphere immediately above the SBL,

$$N^2 = \beta \frac{\partial \theta_v}{\partial z} \quad \text{at} \quad h < z < 2h. \tag{8}$$

N is evidently unimportant for the nocturnal SBLs. The latter develop on the background of well-mixed residual layers, which are near-neutrally stratified. Then N given by Eq. (8) is quite small. From the physical point of view, residual layers separate the nocturnal SBLs from the stably stratified free atmosphere. This prevents vertical propagation of internal waves and makes the SBL turbulence essentially local.

In contrast, long-lived SBLs typical of persisting stable stratification are in immediate contact with the free atmosphere. Then, if the free-flow stability is strong enough, the

SBL turbulence becomes essentially non-local due to wave-induced vertical fluxes of the kinetic energy and the squared buoyancy fluctuations. Zilitinkevich and Calanca (2000) and Zilitinkevich (2001) have shown that N becomes an important governing parameter for this type of SBLs. It is only natural to expect that the critical Richardson numbers also depend on N.

For the surface-layer Richardson number this is already demonstrated by Zilitinkevich et al. (2001b). Moreover, a pronounced dependence on N immediately follows from recent analysis of the finite-difference Richardson number Ri_{Fc}. Thus Vogelezang and Holtslag (1996) reported that Ri_{Fc} for nocturnal SBLs lie in the interval 018 – 0.22 (depending the choice of values of z_1 and z_2), and for slightly stable well-mixed boundary layers, in the interval 0.23 – 0.32. This difference can be very naturally attributed to essentially different typical values of N in the two cases: N close to zero for shallow nocturnal SBLs (capped by residual layers) and N of order 10^{-2} s^{-1} for much deeper well-mixed SBLs (immediately bordering the free atmosphere).

Figure 2 based on the Cabauw data set (the same data as in Figure 1) presents empirical dependencies of the critical bulk Richardson number Ri_{Bc} on the free-flow stability, employing two alternative dimensionless arguments,

$$P = \frac{N}{|f|} \quad \text{and} \quad Q = \frac{NU}{\beta \Delta \theta_v}, \tag{9}$$

of which P characterises comparative roles of the free-flow stratification and earth's rotation and Q, comparative roles of the free flow and the SBL stratification. The Coriolis parameter is fixed, $f = 1.15 \cdot 10^{-4}$ s^{-1}, so that Figure 2a showing $\mathrm{Ri}_{Bc}(P)$ presents an uncombed effect of N. In spite of the wide spread of data, both empirical plots $\mathrm{Ri}_{Bc}(P)$ in Figure 2a and $\mathrm{Ri}_{Bc}(Q)$ in Figure 2b suggest that the effect is significant. The linear- and the power-regression approximations, $\mathrm{Ri}_{Bc} \approx 0.1371 + 0.0024P$ and $\mathrm{Ri}_{Bc} \approx 0.0598 M^{-0.7775}$ are shown by solid lines in Figures 2a and 2b, respectively. For practical use, the first one is recommended, namely,

$$\mathrm{Ri}_{Bc} \approx 0.1371 + 0.0024 \frac{N}{|f|}. \tag{10}$$

The benefit of the use of N-dependent Ri_{Bc} in the bulk Richardson number method is demonstrated in Figure 3. The correlation coefficient between the calculated and the observed SBL heights, h_E and h_{SBL}, increases from 0.56 in the standard version of the method (with $\mathrm{Ri}_{Bc} = 0.25$) to 0.62 in the improved version based on Eq. (10).

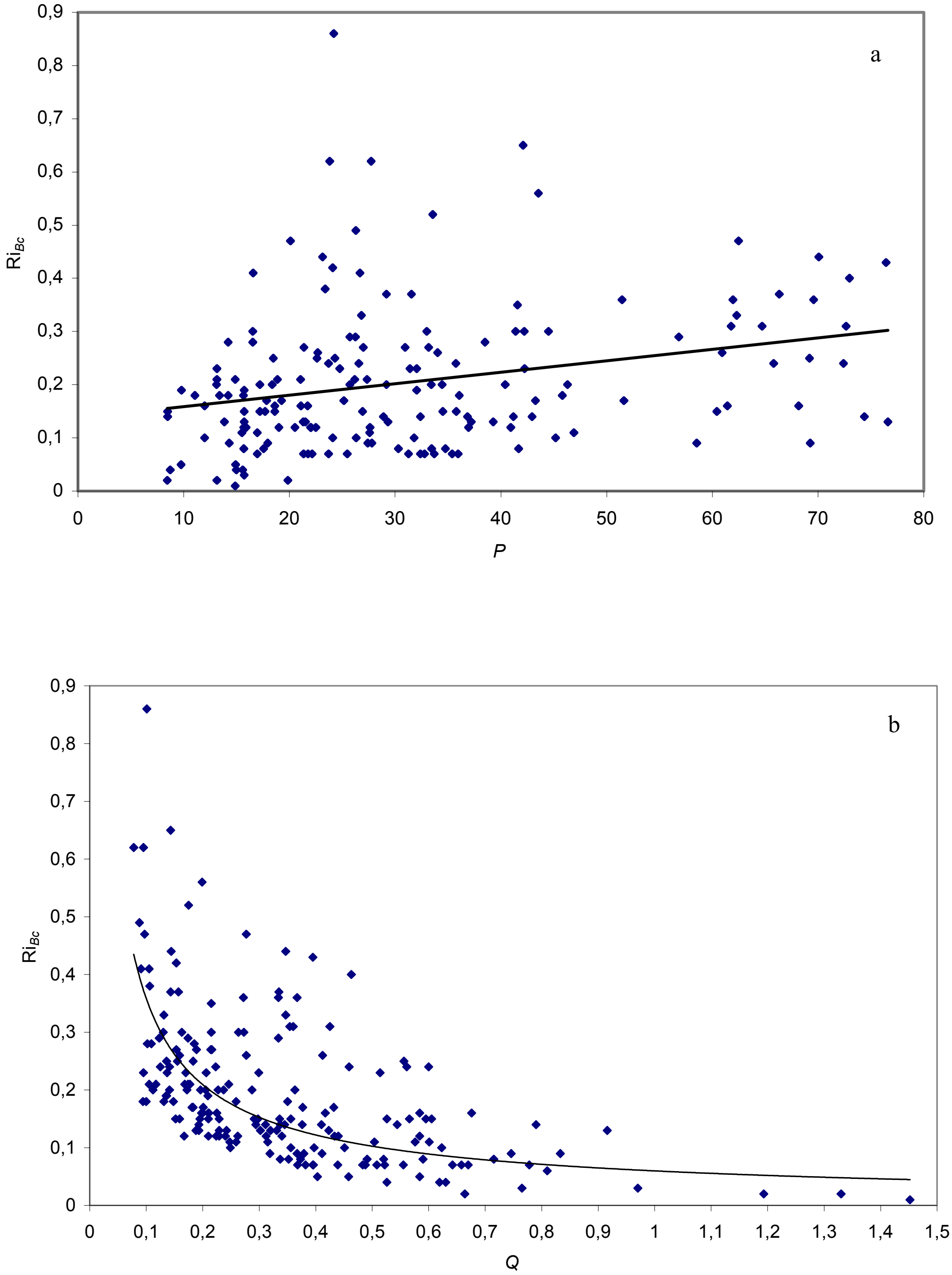
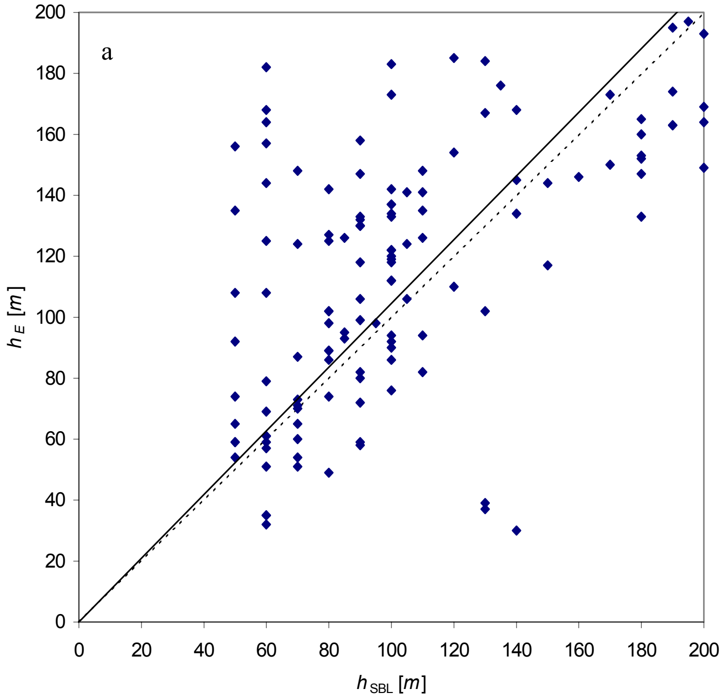


Figure 2. Empirical dependencies of the critical bulk Richardson number Ri_{Bc} on the dimensionless parameters $P = N / |f|$ and $Q = NU / \beta\Delta\theta_v$ involving the free-flow Brunt-Väisälä frequency *N*, after Cabauw data: (a) $\mathrm{Ri}_{Bc}(P)$ and (b) $\mathrm{Ri}_{Bc}(Q)$.



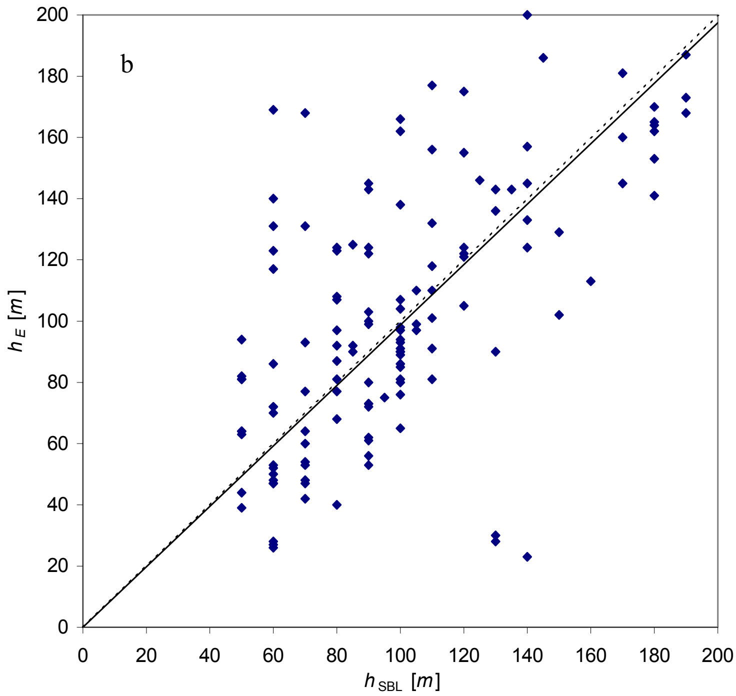


Figure 3. Comparisons of the SBL heights, h_{SBL} deduced from the Cabauw data and h_E estimated through critical bulk Richardson numbers: (a) standard $\mathrm{Ri}_{Bc} = 0.25$, the linear regression line is $h_E = 1.045 h_{SBL}$; (b) N-dependent $\mathrm{Ri}_{Bc} = 0.1371+0.0024N/|f|$, the linear regression line is $h_E = 0.987 h_{SBL}$.

2.5. Surface roughness

One more parameter evidently overlooked in the standard Richardson number methods is the surface roughness length, z_{0u}. Common sense guides us to suppose that the level of turbulent mixing in the SBLs with equal U and $\Delta\theta_v$ should be higher over the surface with the higher roughness. Accordingly the SBL height should also be higher. In terms of the bulk Richardson number, Ri_{Bc}, this reasoning suggests that Ri_{Bc} should increase with increasing z_{0u}. A traditional dimensionless argument involving z_{0u} is the surface Rossby number $\mathrm{Ro}_s = U/|f|z_{0u}$. The dependence of Ri_{Bc} on Ro_s presented in Figure 4 does not contradict the above expectation (remember, all data are taken from Cabauw, so that differences in Ro_s reflect differences in the wind speed rather than in z_{0u}). Anyhow, the linear-regression approximation of data points in Figure 4 reads

$$\mathrm{Ri}_{Bc} \approx 0.2586 - 10^{-7}\frac{U}{|f|z_{0u}}. \tag{11}$$

All things considered, the critical Richardson number methods can be recommended only for rough, order-of-magnitude estimates of the SBL height. More advanced SBL height formulations are discussed below.

3. Multi-limit SBL height equations

3.1 Diagnostic equations

Zilitinkevich and Mironov (1996) employed the turbulent kinetic energy (TKE) budget equation to derive a diagnostic multi-limit equation for the SBL height,

$$\left(\frac{fh_E}{C_n u_*}\right)^2 + \frac{h_E}{C_s L} + \frac{Nh_E}{C_i u_*} + \frac{|fB_s|^{1/2} h_E}{C_{sr} u_*^2} + \frac{|Nf|^{1/2} h_E}{C_{ir} u_*} = 1, \tag{12}$$

where C_n , C_s , C_i, C_{sr} and C_{ir} are dimensionless constants. A week point in the derivation was the use of a rather uncertain parameterisation of the energy dissipation.

At the same time Eq. (12) looks quite attractive. It includes five known length scales. Three of them account for rotation, namely, $h_E \propto u_*/|f|$ for the neutral Ekman layer (Rossby & Montgomery, 1935), $h_E \propto u_*^2/|fB_s|^{1/2}$ and $h_E \propto u_*/|fN|^{1/2}$ for the Ekman layers affected by the surface buoyancy flux (Zilitinkevich, 1972) and the free-flow stability (Pollard et al., 1973), respectively. Two more length scales, L and u_*/N, appropriate for non-rotating stably stratified flows, were employed as the SBL depth scales by Kitaigorodskii (1960) and Kitaigorodskii and Joffre (1988). Zilitinkevich and Mironov (1996) obtained provisional estimations of the above dimensionless constants, $C_n \approx 0.5$, $C_s \approx 10$, $C_i \approx 20$, $C_{sr} \approx 1.0$, and $C_{ir} \approx 1.7$, using data from field measurements and large eddy simulations (LESs).

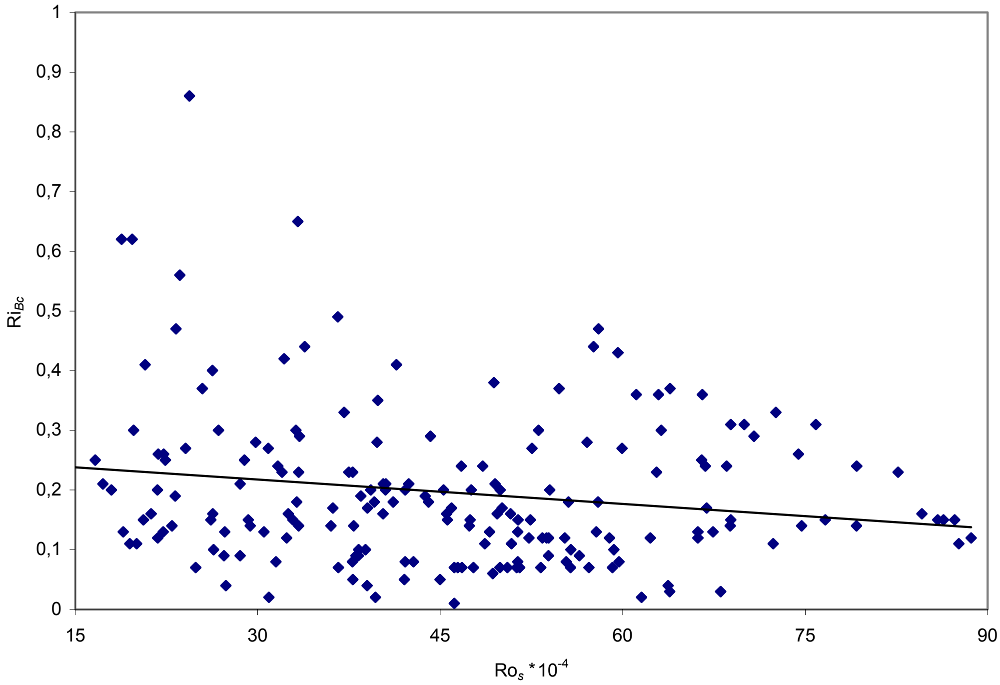


Figure 4. Empirical dependence of the critical bulk Richardson number Ri_{Bc} on the surface Rossby number, $\mathrm{Ro}_s = U/|f|z_{0u}$, after Cabauw data. The linear regression line is $Ri_{Bc} = 0.2586 - 10^{-7}(U/|f|z_{0u})$.

In recent years, Eq. (12) became widely used in air pollution modelling (e.g., Robertson et al., 1996; Brandt, 1998; Linqvist, 1999; Baklanov, 1999; SUBMESO, METPRO and CTDMPLUS models). This deserves further theoretical discussion and empirical verification.

In Figure 5a, the SBL heights, h_E, calculated after Eq. (12) are compared with the actual SBL heights, h_{SBL}, deduced from the Cabauw data set. The correlation can be improved by refinement of empirical constants but only slightly. Thus taking C_n = 0.3 instead of 0.5, Eq. (12) gives higher correlation coefficient and lower root mean square (RMS) error; however, the bias slightly increases and the SBL height remains underestimated (see Table 3 and Figure 5b).

Table 3. Empirical constant C_n in Eq. (12) after Cabauw data.

C_n	**Bias**	**RMS error**	**Correlation coefficient**
0.5	-33.76	33.84	0.384
0.3	-35.91	32.05	0.387

More detailed empirical evaluation of Eq. (12) is given in Table 4 below.

From the theoretical point of view, the TKE budget underlying Eq. (12) was quite probably oversimplified through neglecting the effect of rotation on the energy dissipation. Moreover, straightforward analysis of the Ekman equations indicates that the scales L and u_* / N, included as principal limits in Eq. (12), are not immediately applicable as the depth scales for turbulent boundary layers in rotating fluids.

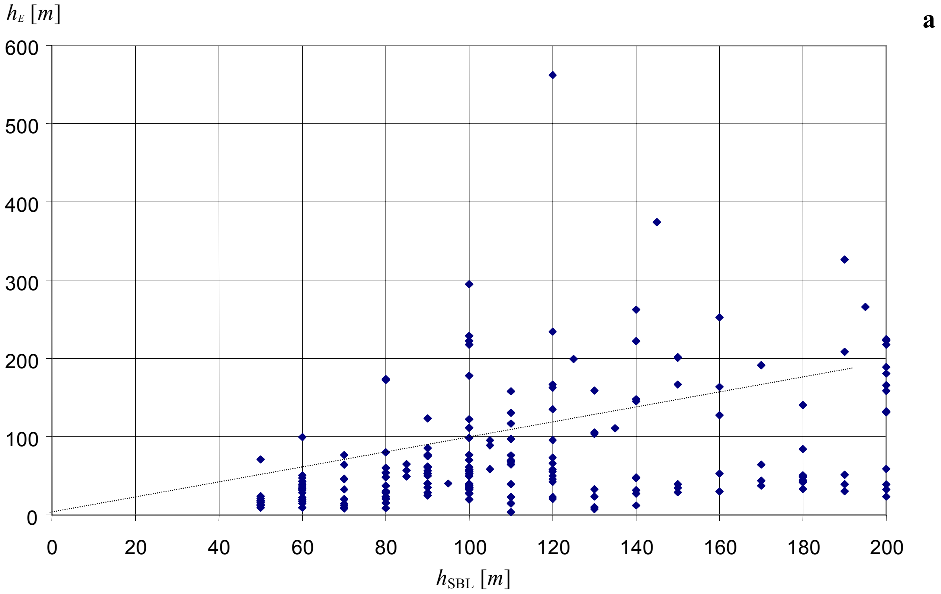
More recently Zilitinkevich et al. (2001a) derived a refined multi-limit Ekman-layer height equation from the momentum equations given a modern formulation for the eddy viscosity (Zilitinkevich, 2001), which implicitly accounted for the TKE budget. The refined equation reads

$$h_E = \frac{C_R u_*}{|f|}\left[1 + \frac{C_R^2 u_* \left(1 + C_{uN} NL/u_*\right)}{C_S^2 |f| L}\right]^{-1/2}, \tag{13}$$

where C_R, C_S, C_{uN} are the same type empirical constants as C_n, C_{sr}, C_{ir} in Eq. (12).

3.2. Vertical-motion correction and prognostic equation

Eq. (13) can be extended to account for the synoptic scale vertical motions through the large-scale vertical velocity, w_h, at the SBL upper boundary. Thus, considering a relaxation equation for the actual SBL height (Zilitinkevich et al., 2001a),



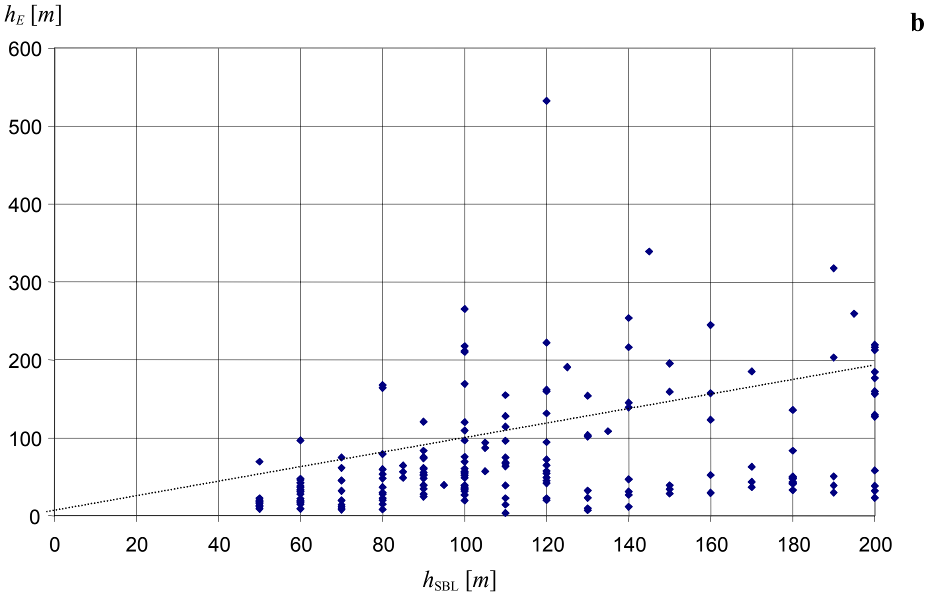


Figure 5. Comparisons of the SBL heights, h_{SBL} deduced from the Cabauw data and h_E after Eq. (12): (a) with $C_n = 0.5$; (b) with $C_n = 0.3$.

$$\frac{\partial h}{\partial t} + \mathbf{V} \cdot \nabla h - w_h = -C_E \, | \, f \, | \, (h - h_E) \, , \tag{14}$$

and keeping only the vertical advection term on the l.h.s. of this equation, a quasi-equilibrium SBL height, h_{QE}, becomes

$$h_{QE} = h_E + \frac{w_h}{C_E \, | \, f \, |} \, , \tag{15}$$

where C_E is one more empirical constant. With C_E ~1 (Zilitinkevich et al., 2001a), the correction term could be as large as ± 100m, which explains quite uncertain performance of any equilibrium SBL height formulations.

Eqs. (13) and (15) are not applicable to non-rotating Equatorial SBLs. When *f* tends to zero, Eq. (14) predicts unlimited growth of the SBL height. However, Equatorial SBLs never live very long. Then the SBL height becomes limited due to the finite periods of the SBL development on the background of the damping effect of capping inversions. Accordingly a climatological upper limit $h \le h_*$ should be put on the SBL height.

Deardorff (1972) was probably the first who employed this idea in a practically oriented boundary-layer height parameterisation. He specified h_* as the height of the tropopause (~15 km). As established by Larsen (2000), climatological upper limits for the boundary-layer height are always much less than 15 km and differ over different geographical sites (e.g., h_* = 3 km for Denmark).

Interpolation between reciprocals of h_{QE} and h_* provides a "corrected quasi-equilibrium SBL height" h_{CQE},

$$\frac{1}{h_{CQE}} = \frac{1}{h_{QE}} + \frac{1}{h_*} \, , \tag{16}$$

where the second term on the r.h.s. is nearly always negligible except for the Equatorial region.

Figure 6 shows a scatter diagram for theoretical versus observed SBL heights (h_{CQE} versus h_{SBL}). Here, h_{SBL} is deduced from the Cabauw data; whereas h_{CQE} is calculated after Eqs. (13), (15), (16) with empirical constants C_R = 0.4, C_S = 0.75 and C_{uN} = 0.25 recommended by Zilitinkevich et al. (2001a). The constant C_E = 1 was not requested as the vertical advection term was not included in calculations. As Figures 5 and 6 suggest, h_{CQE} after Eqs. (13), (15), (16) is a better approximation than h_E after Eq. (12).

Qualitatively these two formulations are evaluated in Table 4 based on the Cabauw data. The table presents the bias (m), the RMS error (m) and the correlation coefficient for h_{CQE} after Eqs. (13), (15), (16), for h_E after Eq. (12) and for some other commonly

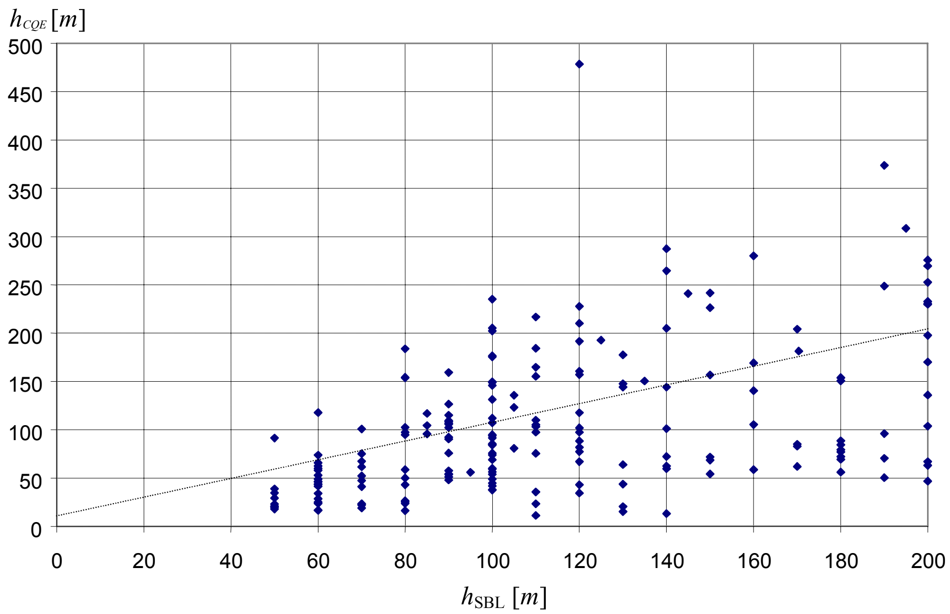


Figure 6. Comparisons of the SBL heights, h_{SBL} deduced from the Cabauw data and h_{CQE} after Eqs. (13), (15), (16) with $C_R = 0.4$, $C_S = 0.75$, $C_{uN} = 0.25$.

used diagnostic SBL height equations. The basic scales given in this table are "RM" (Rossby and Montgomery, 1935) $u_* / |f|$ and "Z" (Zilitinkevich, 1972) $u_*^2 |fB_s|^{-1/2}$. Purely empirical formulas are not based on physical scales.

Table 4. Empirical evaluation of different SBL height equations

Reference	Basic scale	SBL height equation	Bias	RMS error	Corre-lation
Benkley & Schulman, 1979	None	$h = 125\, u_{10}$	208	264	0.48
Arya, 1981	Z	$h = 0.42\, u_*^2 \| fB_s \|^{-1/2} + 29.3$	64.0	218	0.27
Arya, 1981	RM	$h = 0.089\, u_* / \| f \| + 85.1$	103	86.3	0.48
Mahrt, 1982	RM	$h = 0.06\, u_* / \| f \|$	-24.4	18	0.48
Niewstadt, 1984	None	$h = 28\, u_{10}^{3/2}$	6.27	13.9	0.48
Niewstadt, 1984	Z	$h = 0.4\, u_*^2 \| fB_s \|^{-1/2}$	24.4	173	0.27
Zilitinkevich & Mironov, 1996	Multi-scale	$h = h_E$ after Eq. (12)	-33.8	33.8	0.38
Zilitinkevich et al., 2001a	Multi-scale	$h = h_{CQE}$ after Eqs. (13), (15) and (16) with $w_h = 0$	6.21	19.2	0.60

It is worth mentioning that h_{CQE} - Eqs. (13), (15), (16) performs quite well in the cases with strong free-flow stability. Here, the bias and the RMS error become 3.8 m and 18.7 m, respectively.

It follows that h_{CQE} after Eqs. (13), (15), (16) is a reasonable diagnostic SBL height formulation. It is recommended for use within 1-D models. Within 3-D models, the SBL height, *h*, can be calculated more accurately on the basis of Eq. (14). Employing h_{CQE} instead of h_E and accounting for the sub-grid scale horizontal motions through the horizontal diffusivity K_h, Eq. (14) becomes

$$\frac{\partial h}{\partial t} + \mathbf{V} \cdot \nabla h = -C_E |f| (h - h_{CQE}) + K_h \nabla^2 h, \qquad (17)$$

where $\mathbf{V} = (u, v)$ is the horizontal velocity vector. The vertical advection term $(-w_h)$ is included in h_{CQE}.

4. Concluding remarks

The SBL critical bulk Richardson number, Ri$_{Bc}$, is not a constant. It evidently increases with increasing free flow stability and very probably depends on the surface roughness length and the Coriolis parameter. The Richardson-number-based calculation techniques can be recommended only for rough estimates of the SBL height.

For more accurate SBL height calculations within 1-D and 3-D models, respectively, the diagnostic formulation h_{CQE} - Eqs. (13), (15), (16) and the prognostic formulation h - Eq. (17) are recommended.

Acknowledgements

The authors thank Søren Larsen (Risø National Laboratory, Denmark) and George Djolov (University of Venda) for discussions, Bert Holtslag and Daan Vogelezang (KNMI, The Netherlands) for providing the Cabauw data. This work was supported by the SIDA Project "Non-local turbulent transport in weather prediction and air pollution modelling" (South African – Swedish Research Partnership Programme 2000), the NFR Project "Wave-turbulence Meso-scale Dynamics over Complex Terrain: Modelling, Observation and Parameterisation" – Contract NR: G-AA/GU 12471-300, partially the EU project CARTUM – Contract MAS3-CT98-0172 and the Danish Forskningsstyrelsen – START-project No. 44.2207.4422.

References

Anisimova, Ye.P., Porborchaya, L.V. and Speranskaya, A.A.: 1978, 'Velocity profile in the boundary layer of stratified turbulent flow', *Izv. USSR Acad. Sci., Atmospheric and Oceanic Physics*, **14**, 785-788.

Arya, S.P.S.: 1972, 'The critical condition for the maintenance of turbulence in stratified flows', *Quart. J. Roy. Meteor. Soc.* **98**, 269-273.

Arya, S.P.S.: 1981, 'Parameterizing the height of the stable atmospheric boundary layer', *J. Appl. Meteorol*. **20**, 1192-1202.

Andreas, E.L., Claffey, K.J. and Makshtas, A.P.: 2000, 'Low-level atmospheric jets and inversions over the western Weddell sea', *Boundary-Layer Meteorol*. **97**, 459-486.

Baklanov, A.: 1999, 'Modelling of episodes of atmospheric transport and deposition: Hypothetical nuclear accidents in North-West Russia', in *Nuclear Risks, Environmental and Development Cooperation in the North of Europe*, CERUM, University of Umeå, Sweden, pp. 65-80.

Baltink, H.K. and Holtslag, A.A.M.: 1997, 'A comparison of boundary-layer heights inferred from windprofiler backscatter profiles with diagnostic calculations using regional model forecasts', in S.-E. Gryning (ed.), *EURASAP workshop proceedings on The determination of the mixing height – current progress and problems,* Risø-R-997(EN), pp. 51-54.

Benkley, C.W. and Schulman, L.L.: 1979, 'Estimating mixing depths from historical meteorological data', *J. Appl. Meteorol*. **18**, 772-780.

Brandt, J.: 1998, *Modelling transport, dispersion and deposition of passive tracers from accidental releases*, PhD thesis, RISØ-NERI, Denmark, 307 pp.

Brost, R.A. and Wyngaard, J.C.: 1978, 'A model study of the stably stratified planetary boundary layer', *J. Atmos. Sci*. **35**, 1427-1440.

Brutsaert, W.: 1972, ‘Radiation, evaporation and the maintenance of the turbulence under stable conditions in the lower atmosphere’, *Boundary-Layer Meteorol*. **2**, 309-325.
Businger, J.A., Wyngaard, J.C., Izumi, Y. and Bradley, E.F.: 1971, ‘Flux profile relationships in the atmospheric surface layer’, *J. Atmos. Sci*., **28**, 181-189.
Businger, J.A.: 1973, ‘Turbulent transfer in the atmospheric surface layer’, in D.A. Haugen (ed.), *Chapter 2 in Workshop on Micrometeorology*, American Meteorological Soc., Boston, Mass.
Byzova, N.L., Ivanov, V.N. and Garger, E.K.: 1989, *Turbulence in the boundary layer of the atmosphere*, Leningrad, Gidrometeoizdat, 264 pp.
Deardorff, J.W.: 1972, ‘Parameterization of the planetary boundary layer for use in general circulation models’, *Mon. Wea. Rev*., **100**, 93-106.
Fay, B., Schrodin, R., Jacobsen, I. and Engelbart, D.: 1997, ‘Validation of mixing heights derived from the operational NWP models at the German Weather Service’, in S.-E. Gryning (ed.), *EURASAP workshop proceedings on The determination of the mixing height – current progress and problems,* Risø-R-997(EN), pp. 55-58.
Hanna, S.R.: 1969, ‘The thickness of the planetary boundary layer’, *Atm. Environ*. **3**, 519-536.
Heineman, G. and Rose, L.: 1990, ‘Surface energy balance, parameterisation of boundary-layer height and the application of resistance laws near an Antarctic ice shelf front’, *Boundary-Layer Meteorol*. **51**, 123-158.
Holtslag, A.A.M. and Boville, B.A.: 1993, ‘Local versus nonlocal boundary-layer diffusion in a global climate model’, *J. Climate*, **6**, 1825-1842.
Holtslag, A.A.M., De Bruin, E.I.F. and Pan, H.-L.: 1990, ‘A high resolution air mass transformation model for short range weather forecasting’, *Mon. Wea. Rev*. 118, 1561-1575.
Joffre, S. M.: 1981, *The physics of the mechanically driven atmospheric boundary layer as an example of air-sea ice interaction*, Univ. of Helsinki, Dept. Of Meteorology, Rep. 20, 75 pp.
Kazansky, A. B., and Monin, A. S.: 1960, ‘On the regime of turbulence above the atmospheric surface layer’, *Izvestiya AN SSSR, Geophysical Series*, **1**, 165-168.
King, J. and Turner, J.: 1997, *Antarctic Meteorology and Climatology*, Cambridge University Press.
Kitaigorodskii, S. A.: 1960, ‘On the computation of the thickness of the wind-mixing layer in the ocean’, *Izvestiya AN SSSR, Geophysical Series*, **3**, 425-431.
Kitaigorodskii, S. A., and Joffre, S. M.: 1988, ‘In search of simple scaling for the heights of the stratified atmospheric boundary layer’, *Tellus*, **40A**, 419-443.
Laikhtman, D. L.: 1961, *Physics of the atmospheric boundary layer*. Leningrad, Gidrometeoizdat, 200 pp.
Larsen, S.: 2000, ‘Observations of boundary layer height at high wind speed neutral situations’, Presentation on the *Workshop on Stable Boundary Layers*, Uppsala University, Sweden, 12-14 September 2000.
Linqvist J.: 1999, *En stokastik partikelmodell i ett icke-metriskt koordinatsystem*, FOA report, FOA NBC-defence, Umeå, Sweden, 46 pp.
Mahrt, L.: 1981, ‘Modelling the depth of the stable boundary layer’, *Boundary-Layer Meteorol.* **21**, 3-19.
Mahrt, L.: 1982, ‘Momentum balance of gravity flows’, *J. Atmos. Sci.* **39**, 2701-2711.

Mahrt, L., Heald, R.C., Lenschoow, D.N., Stankow, B.B., Troen, I.: 1979, 'An observational study of the structure of the nocturnal boundary layer', *Boundary-Layer Meteorol*. **17**, 247-264.

Makshtas, A.P., Timachev, V.F. and Andreas, E.L.: 1998, 'Structure of the lower atmospheric layer over ice cover of the Weddell sea', *Russian Meteorology and Hydrology*, **10**, 68-75.

Maryon, R.H. and Best, M.J.: 1992, *'NAME', 'ATMES' and the boundary layer problem*, UK Met. Office (APR) Turbulence and Diffusion Note, No. 204.

Maryon, R.H., Smith, J.B. Conway, B.J. and Goddard, D.M.: 1991, 'The United Kingdom Nuclear Accident Model', *Prog. Nucl. Energy*, **26**, 85-104.

Melgarejo, J.W. and Deardorff, J.W.: 1974, 'Stability functions for the boundary layer resistance laws based upon observed boundary layer heights', *J. Atmos. Sci*. **31**, 1324-1333.

Nieuwstadt, F.T.M.: 1984, 'The turbulent structure of the stable, nocturnal boundary layer', *J. Atmos. Sci*. **41**, 2202-2216.

Pollard, R. T., Rhines, P. B., and Thompson, R. O. R. Y.: 1973, 'The deepening of the wind-mixed layer', *Geophys. Fluid Dyn*. **3**, 381-404.

Robertson, L., Langner, J. and Engardt, M.: 1996, *MATCH - Meso-scale Atmospheric Transport and Chemistry modelling system. Basic transport model description and control experiments with* ^{222}Rn, SMHI RMK No 70, Norrkoping, Sweden.

Rossby, C. G., and Montgomery, R. B.: 1935, 'The layer of frictional influence in wind and ocean currents', *Pap. Phys. Oceanogr. Meteorol*. **3**, No.3, 1-101. (M.I.T. and Woods Hole Oceanogr. Inst.)

Seibert, P., Beyrich, F., Gryning, S.-E., Joffre, S., Rasmussen, A. and Tercier, Ph.: 1998, 'Mixing Height Determination for Dispersion Modelling. Report of Working Group 2', in *COST Action 710 – Final report. Harmonisation in the pre-processing of meteorological data for atmospheric dispersion models*, EUR 18195 EN, 120 pp.

Seibert, P., Beyrich, F., Gryning, S.-E., Joffre, S., Rasmussen, A. and Tercier, Ph.: 2000, 'Review and Intercomparison of Operational Methods for the Determination of the Mixing Height', *Atm. Environ.* **34**, 1001-1027.

Straume, A.G., N'Dri Koffi, E. and Nodop, K.: 1998, 'Dispersion modelling using ensemble forecasts compared to ETEX measurements', *J. Appl. Meteorol*. **37**, 1444-1155.

Sørensen, J.H., Rasmussen, A. and Svensmark, H.: 1996, 'Forecast of Atmospheric Boundary-Layer Height for ETEX Real-time Dispersion Modelling', *Phys. Chem. Earth*, **21**, No. 5-6, 435-439.

Taylor, G.I.: 1931, 'Effects of variation in density on the stability of superimposed streams of fluids', *Proc. R. Soc. (London), Ser. A*, **132**, 499-523.

Troen, I., and Mahrt, L.: 1986, 'A simple model of the atmospheric boundary layer: sensitivity to surface evaporation', *Boundary-Layer Meteorol*. **37**, 129-148.

Van Ulden, A. P., and Wieringa, J.: 1996, 'Atmospheric boundary layer research at Cabauw', *Boundary-Layer Meteorol*. **78**, 39-69.

Vogelezang, D.H.P. and Holtslag, A.A.M.: 1996, 'Evolution and model impacts of the alternative boundary layer formulations', *Boundary-Layer Meteor*. **81**, 245-269.

Webb, E.K.: 1970, 'Profile relationships: the long-linear range and extension to strong stability', *Quart. J. Roy. Meteorol. Soc*. **96**, 67-90.

Wentzel, P.J.: 1983, 'Toward parameterisation of the stable boundary layer', *J. Appl. Meteorol*. **21**, 7-13.

Zeman, O.: 1979, ‘Parameterisation of the dynamics of stable boundary layers and nocturnal jets’, *J. Atmos. Sci*. **36**, 792-804.
Zilitinkevich, S. S.: 1970, *Dynamics of Atmospheric Boundary Layer*, Gidrometeoizdat, Leningrad, 192 pp.
Zilitinkevich, S. S.: 1972, ‘On the determination of the height of the Ekman boundary layer’, *Boundary-Layer Meteorol*. **3**, 141-145.
Zilitinkevich, S. S.: 2001, ‘Third-order transport due to internal waves and non-local turbulence in the stably stratified surface layer’, Submitted to *Quart. J. Roy. Meteorol. Soc.*
Zilitinkevich, S., Baklanov, A., Rost, J., Smedman, A.-S., Lykosov, V. and Calanca, P.: 2001a, ‘Diagnostic and prognostic equations for the depth of the stably stratified Ekman boundary layer’, *Quart. J. Roy. Meteorol. Soc.* (accepted).
Zilitinkevich, S., and Calanca, P.: 2000, ‘An extended similarity-theory formulation for the stably stratified atmospheric surface layer’, *Quart. J. Roy. Meteorol. Soc.* **126**, 1970-1985.
Zilitinkevich, S., and Mironov, D. V.: 1996, ‘A multi-limit formulation for the equilibrium depth of a stably stratified boundary layer’, *Boundary-Layer Meteorol.* 81, 325-351.
Zilitinkevich, S. S., Perov, V. and King, J.: 2001b, ‘Near-surface turbulent fluxes in stable stratification: calculation techniques for use in general circulation models’, Submitted to *Quart. J. Roy. Meteorol. Soc.*

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