DMI Report 21-26
Extreme Value Analysis of Daily Temperature Records in Denmark, 2001-2020

Final scientific report of the 2020 National Centre for Climate Research Work Package 2.2.1, KlimaNU

Rasmus Stoltze Hansen, Peter Thejll & Mikael Scharling

København 2021

Danmarks
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Colophon

Series Title
DMI Report 21-26

Title
Extreme Value Analysis of Daily Temperature Records in Denmark, 2001-2020

Sub Title
Final scientific report of the 2020 National Centre for Climate Research Work Package 2.2.1 KlimaNU

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Publisher
Danmarks Meteorologiske Institut (Danish Meteorological Institute)

Language
English

Keywords
Extreme Value Analysis, Return Period, Temperature

Digital ISBN
978-87-7478-700-6

ISSN
2445-9127

Version date
15th January 2021

Link to website
www.dmi.dk

Danmarks
Meteorologiske
Institut

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1. Abstract

This study was carried out by the Danish Meteorological Institute (DMI) for the Danish National Centre for Climate Research (NCKF). The study investigated extreme temperature events in Denmark in the period 2001-2020 based on data from 60 weather stations. The project was developed as a pilot-study based on daily temperature records of a gridded dataset (1x1 km) over Denmark in order to standardize classification and description of extreme weather situations regarding temperature.
In the study 2, 5, 10, 20, 40 and 100-year return periods were calculated on a monthly basis for minimum, mean and maximum temperature. Both low and high extreme were calculated for all three parameters. A Generalized Extreme Value analysis was performed and initially showed that when increasing the length of the input data set, the analysis becomes more robust. Likewise did the analysis show that the uncertainty bands grew wider with increasing return period for all three parameters. Though, a two-sided Kolmogorov-Smirnov test showed that data did not meet the requirement of stationarity during the period. Therefore it is suggested that a follow-up study is carried out using extreme value modeling based on co-variate modelling as a remedy. Additionally it is suggested that the analysis is repeated with more data (e.g. 30 years).

2. Resumé

Dette projekt er udarbejdet af Danmarks Meteorologiske Institut (DMI) under det Nationale Center for Klimaforskning (NCKF). Projektet undersøgte ekstreme temperaturhændelser i Danmark på baggrund af data fra 2001-2020 baseret på 60 vejrstationer. Projektet blev gennemført som et pilot-studie, udført på daglige temperatur data i et griddet datasæt (1x1 km) dækkende hele Danmark. Hensigten var at tilvejebringe en standardiseret klassifikation og beskrivelse af ekstreme temperaturhændelser i Danmark.

Der blev beregnet 2, 5, 10, 20, 40 og 100-års hændelser på månedsbasis for minimum-, middel- og maksimumtemperatur. Både lave og høje ekstremer blev beregnet for alle tre parametre. En Generalized Extreme Value analyse blev udført og viste at robustheden af analysen forbedres med øget datamængde. Ligeledes viste analysen, som forventet, at usikkerhedsintervaller øgedes ved længere returperiode for alle tre parametre. Dog viste en to-sidet Komogorov-Smirnov test, at data ikke opfyldte kravene om stationaritet for den anvendte analyse. Derfor foreslås det at modelleringen genkøres med co-variater som forklarende faktor. Dertil foreslås det at analysen genkøres på baggrund af et datasæt (fx 30 år).

3. Introduction

The Danish National Centre for Climate Research (Nationalt Center for Klimaforskning, NCKF) has completed its first year in 2020. It has been a source of funding for the Danish Meteorological Institute and collaborators for climate change related research during this year. The 18 work packages fall under 4 general themes:

1. Arctic and Antarctic Research
2. Climate change in the near future
3. Use of climate data
4. Support for the IPCC

This report is prepared under work package KlimaNU v. 2.2.1 for National Center for Climate Research (NCFK).
KlimaNU v. 2.2.1 is developed in order to standardize classification and description of extreme weather situations regarding temperature. The study will improve communication and/or reporting both before, during and after extreme or dangerous weather situations.

KlimaNU v.2.2.1 is also a prototype of a refined and continued project to come, including Extreme Value Analysis (EVA) of other parameters, such as precipitation, wind and sunshine. It is furthermore the intention that the development will also include an improvement of the spatial resolution of data from national to municipality level. This continued project is set to be developed during 2021.

4. Background

Extreme events can be defined in different ways, depending on the asset in terms of users and purposes.
From a user's perspective extremes may involve risks to health, finances and social impact.

The KlimaNU project is a pilot-study on extreme temperature events in Denmark, carried out on daily temperature records during 2001-2020 (minimum, mean and maximum temperature). The purpose of the study is to develop a standard for classification, description and presentation of extreme weather situations relevant for Danish weather conditions.

With this study we have investigated the occurrence of extreme temperature events in Denmark and attempted to display their normality based on statistical analysis.

5. Extreme Value Analysis

EVA is a statistical technique that allows us to predict the probability of extreme values occurring in a data set. This could be extreme temperatures or periods of cold or warm spell.

EVA statistics are primarily used in order to quantify the stochastic behavior of extreme values (small or large). In many academic fields extreme value theory is applied, such as meteorology, hydrology and ocean wave modelling.

EVA can characterize the risk of specific natural hazards connected to the investigated parameters and ultimately assist in developing and building resilient societal infrastructures.

Characterization of extreme events depends on the asset you want to protect – thus, different infrastructural assets have different thresholds. EVA can be used as an assisting tool in order to determine and establish the proper decisions associated with infrastructural designs of many kinds. These thresholds are commonly referred to as a *return level* of the weather variable, associated with a *return period.*

It is often used to determine the probability of a certain event such as the 10, 50 or 100 year return periods for a given weather parameter, e.g. temperature.

A return period of 100 years refers to the average expected occurrence of 1 time every 100 years of that extreme return level event, for example a certain maximum temperature.

The classic example is an asset that may not function beyond a certain temperature, and it can then be investigated what the return period of a certain return level is.
Thus, EVA can be used for a wide variety of societal groups ranging from coastal protection and flood management to sewer dimensioning and building construction.

Observed weather data provides the baseline for this EVA and in most cases it is attempted to extrapolate the observed data period beyond the limits of what have been observed historically. The longer the observed data period is the more accurate the generated estimates of return levels are.

Analyzing the statistics of extreme events can be challenging due to multiple reasons. Often extremes are characterized by outliers, which implies that there are few example of such events in the observed data. Therefore, estimation of the uncertainty of calculated return levels are needed, and are often used directly in planning efforts as the uncertainty on an estimate determines to what level the estimate itself is useful, given various applications.

6. Data material

The data applied for this study originate from DMI's observation network which currently includes approximately 60 weather stations.
A detailed overview of the stations can be found in DMI Technical Report 13-13 (DMI, 2013).

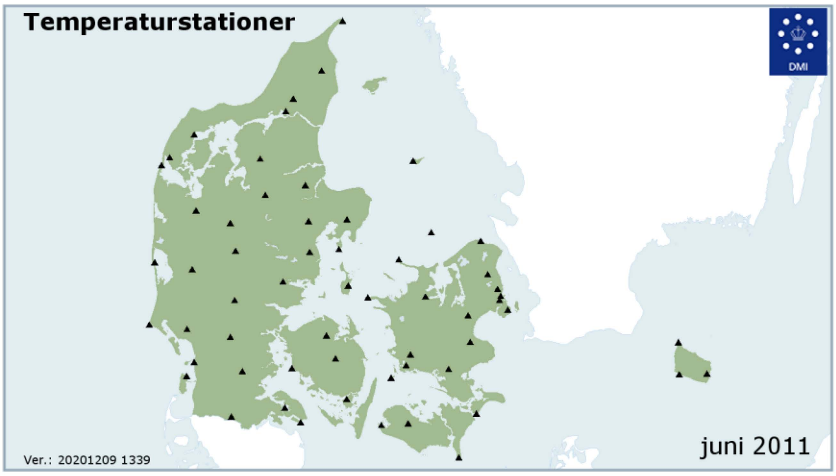

Figure 1. Maps of weather stations in Denmark by June 2011.

DMI's observation network consists of 60 automatic weather stations. All stations are placed and installed so they to the greatest extent meet the World Meteorological Organization's (WMO) recommended guidelines for installation of meteorological observation instrumentation (WMO, 2018).

6.1 Basis Data DK

From DMI's observation network raw data are transmitted to an observation database and after reprocessing, the data are sent to a climate database as hourly climate data (refered to as Basisdata).

Basisdata are values with the lowest possible time resolution meeting the following criteria:

- Values are station data (observed or interpolated)
- Values are quality controlled
- Time series are complete (flawed and missing values have been replaced with interpolated values)
- Data only exists from a station from start-date till termination-date
- Time resolution is equal to or less than 24 hours

From *basisdata* daily, monthly and yearly values are calculated.

6.2 In-put data

For this study the following data has been used in the analysis:

Table 1. Input data for EVA

Parameter	Spatial resolution	Temporal resolution	Period start	Period ending
Minimum temperature	Denmark	Daily	2001	2020 (Oct)
Mean temperature	Denmark	Daily	2001	2020 (Oct)
Maximum temperature	Denmark	Daily	2001	2020 (Oct)

Daily mean temperature is the average of all hourly mean temperatures grid cells during the day and minimum and maximum temperature is respectively the lowest and the highest temperature measured during the day.

The calculation of daily values follows the national calendar day, which is changed twice during the year with the transition from normal-time to daylight saving time and back. This means that one normal day consists of 24 hours, except two days every year. With the transition from normal-time to daylight saving-time the day length is 23 hours and the transition from daylight saving-time back to normal-time the day length is 25 hours.

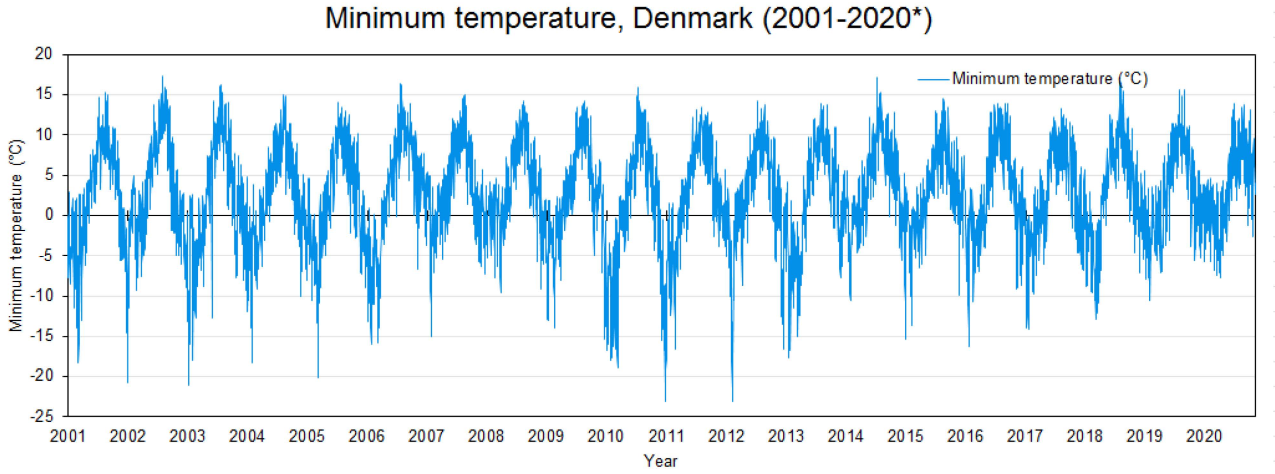

Figure 2. Daily minimum temperature for Denmark, 2001-2020. *2020 including October.

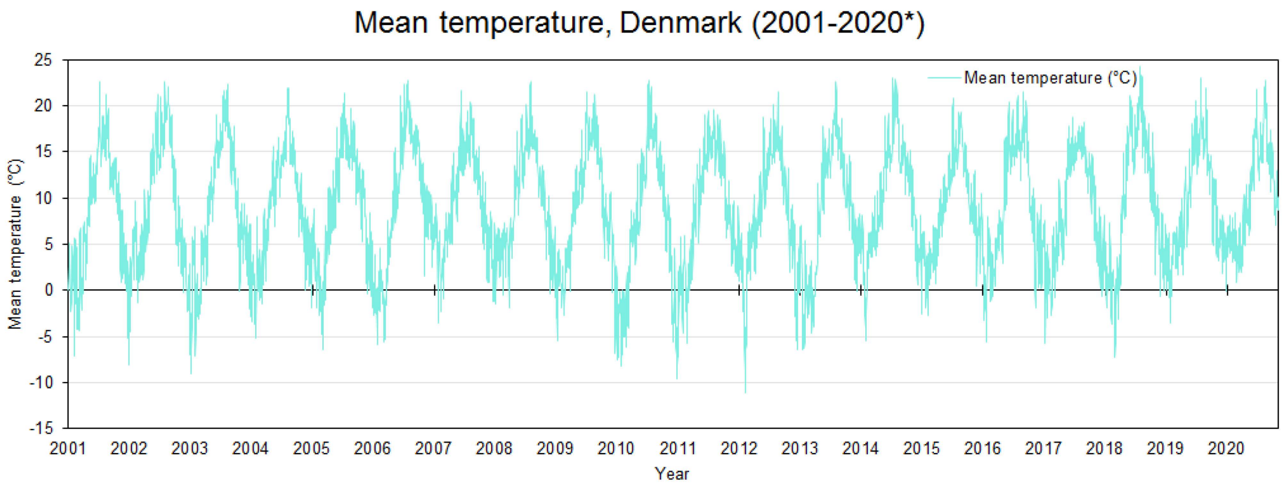

Figure 3. Daily mean temperature for Denmark, 2001-2020. *2020 including October.

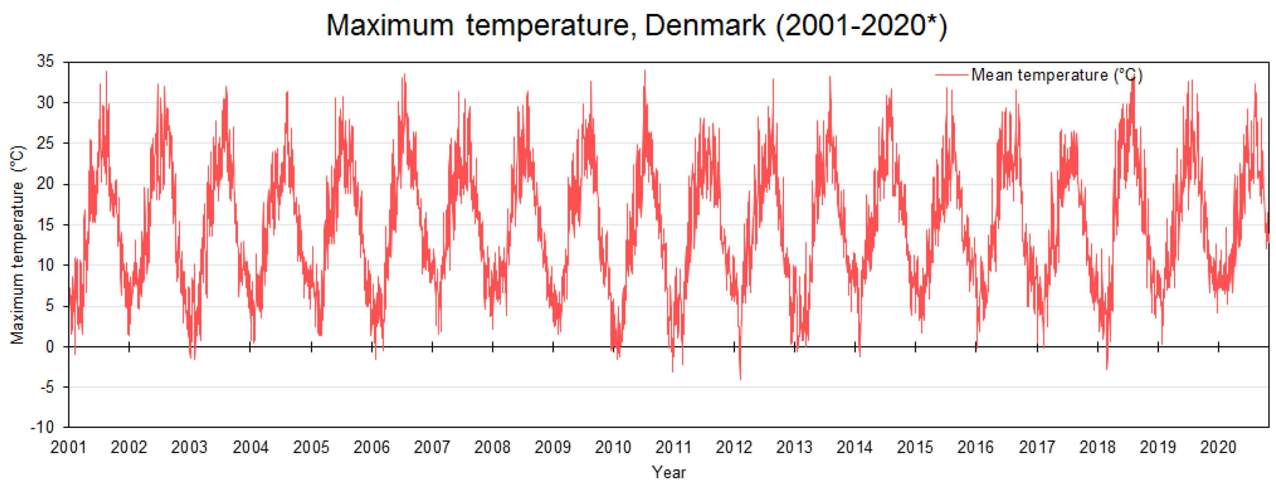

Figure 4. Daily maximum temperature for Denmark, 2001-2020. *2020 including October.

To provide an overview of the extremes within the dataset, table 2 shows the high and low extremes occurring for all three parameters:

Table 2. Minimum and maximum value for all three parameter (minimum, mean and maximum temperature) in Denmark for all 12 calendar months in the data input period 2001-2020.

Month	**Minimum Temperature (°C)**		**Mean Temperature (°C)**		**Maximum Temperature (°C)**	
	Min	*Max*	*Min*	*Max*	*Min*	*Max*
Jan	-21	5,6	-9,0	9,4	-1,6	12,4
Feb	-23,1	4,9	-11,1	9,6	-4,1	15,8
Mar	-20,2	5,8	-6,4	12,1	-1,3	21,5
Apr	-8,9	7,4	0,6	14,9	6	26,7
May	-4,5	12,9	4,7	21,1	10,3	30,7
Jun	-0,1	14,2	8,8	21,9	14,7	32,7
Jul	1,8	17,2	11,9	24,3	16,8	34,1
Aug	1,3	17,4	11,8	23,2	17,2	33,9
Sep	-2,2	14,1	8,5	20,6	13,3	29,9
Oct	-7,8	13,2	-0,1	17,1	6,2	26,9
Nov	-11,5	10,2	-5,6	13,2	-0,2	16,7
Dec	-23	7,8	-9,6	10,6	-3,1	14,2

6.3 Quality Control

Raw observations from national synoptic weather stations may contain various errors and data outages. It is a slow but very important task to identify these errors before calculating extreme values. For this study all climate data in the period from January 1st 2001 to October 31, 2020 has gone through quality control

All data have been quality controlled at multiple levels:

1. Spatial controls on a daily, monthly and yearly level, performed on a visual inspection of interpolated maps with station data plotted on
2. Visual control of the stations based on time plot

When potential errors are identified they are investigated closely. For example by looking at raw observation data or including other data sources such as nearby stations or other parameters which can help to conclude whether the data must be excluded.

Typical errors in the observations are unrealistic high or low outliers. Furthermore certain stations are excluded for longer periods, if the station observes faulty data climatologically spoken.

When errors are removed from the dataset, values are replaced by interpolated values derived from surrounding data.

6.4 Interpolation Algorithm – Grid data

Based on the quality controlled *Basisdata* all station data are interpolated into a 1x1 km grid net, covering all of Denmark's area.

The overall factor that generally spoken has the largest influence on the local climate in Denmark is the distance to the sea. The uneven station coverage of Denmark's area represents a challenge if using a classic non-weighted interpolation. Areas with bad coverage are in risk of being affected by stations located in an area that climatologically seen is not representative for the point of interpolation.

Therefore a modified inverse-distance algorithm is applied, where the value in each grid cell depends on the values of the nearest surrounding station found in 8 sectors. The stations are weighted in relation to the distance and climatological comparability (distance to sea) to the grid.

The spatial interpolation algorithm consists of three steps, which are explained below:

1) Station selection
2) Applying interpolation algorithm
3) Filtering

6.4.1 Station Selection

Each grid cell value is calculated from weighted station values. A grid cell is not represented by every station equally. Station selection is needed since some stations may be located in different climate or add undesired effects to the grid cell value.

The algorithm selects the nearest station in 8 sectors.

In the following the station-selection method is illustrated for the case shown in Figure 5. The green triangles illustrate stations and the red square is the grid cell to be calculated. The algorithm creates four lines passing through the center of the grid cell, creating eight sectors (ENE, NNE, NNW, WNW, WSW, SSW, SSE, ESE) and find the nearest station in each sector – aquamarine triangles in Figure 5 right panel).

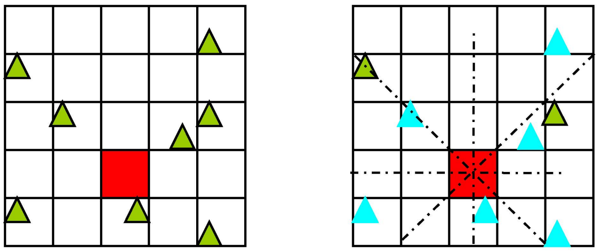

Figure 5. *Left panel*: Station selection case - Green are stations, red grid point to be calculated
Right panel: Station selection case - Using eight sectors

6.4.2 Applying Interpolation Algorithm

When the stations to be used for a grid cell interpolation are determined, the value can be calculated in two different ways. Some meteorological parameters depend on coast/inland distribution, while others do not, furthermore the density of the stations can vary for different parameters.
In this study, where only temperature data was used, we show the interpolation including the coastal/inland climate ratio.

$$v = \frac{\sum_{i=0}^{N}\left(k_j \frac{1}{r_i^a} * s_i\right)}{\sum_{i=0}^{N}\left(k_j \frac{1}{r_i^a}\right)} \quad (1)$$

Where table 3 below contains the parameter explanation:

Table 3a. Parameter description of Equation 1 (Interpolation algorithm including coastal/inland distribution.

Symbol	Description
v	value of the grid point
r_i	distance to station *i*
a	empiric exponent
i	station index
N	number of stations
s_i	value of station i
k_j	Attenuation coast/inland distribution between station and the grid cell $k_j = \frac{100-\lvert k_v - k_i\rvert}{\sum_{i=0}^{N}(100-k_v-k_i)}$
k_v	Coast/inland distribution for the grid cell
k_i	Coast/inland distribution for the i[th] station

The coast- /inland climate distribution follows figure 6 below:

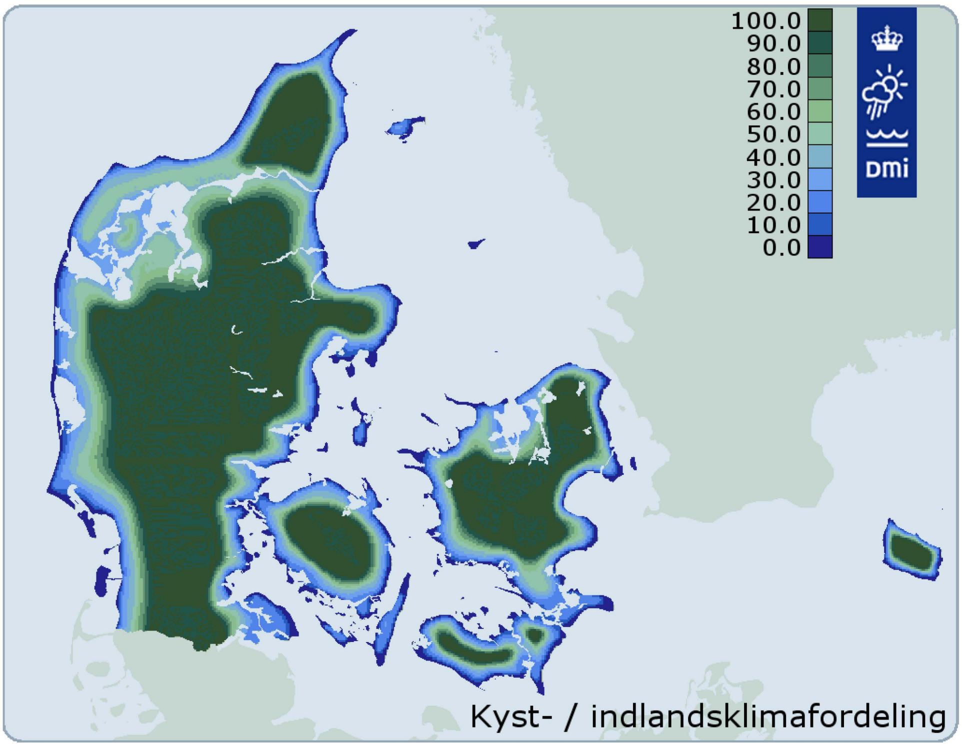

Figure 6: Empirical* coast- /inland climate distribution in Denmark used in the interpolation algorithm (*Defined by DMI)

6.4.2 Filtering

After interpolation algorithms has been run the image is processed by a Gaussian filter (coefficients shown in Figure 7), in order to remove sharp edges.

2	7	12	7	2
7	31	52	31	7
12	52	127	52	12
7	31	52	31	7
2	7	12	7	2

Figure 7. Gaussian filter coefficients

The filtering is done by making a convolution of the interpolation result with the mask shown in Figure 7. The convolution is slightly modified in order to have a reasonable result at the edges of islands (e.g. is there is a missing value on the island edge the weighting is done only on the existing values).

Near the location of the stations, the effects from the raw interpolation are desired, while far away from the location of stations the filtered (blurred) information is desired.

To achieve the above stated effect a linear function is used to weigh the raw interpolated data and the filtered data.

$$v = w1 * A + w2 * B \tag{2}$$

Where:

Table 3b. Parameter description of Equation 2 (linear equation used to weigh interpolated and filtered data)

Symbol	**Description**
w1	$w1 = \begin{cases} 0 \leq r \leq 10000m & 1 - 1\frac{1}{10000m}r \\ 10000m < r & 0 \end{cases}$
w2	1- *w*1
A	Interpolated values
B	Filtered interpolated values
r	Distance to nearest station [m]
v	Value of grid point

In short the above uses a combination of pure interpolation and filtered interpolation in a distance up to 10km from a station position. The grid points that are further away from a station than 10km are based purely on the filtered data.

6.4.3 Validation

The validation is performed in order to find out how accurate the spatial interpolation is compared to in-situ measurements.

In Figure 8 the procedures for using full-cross validations on a set of data are shown.

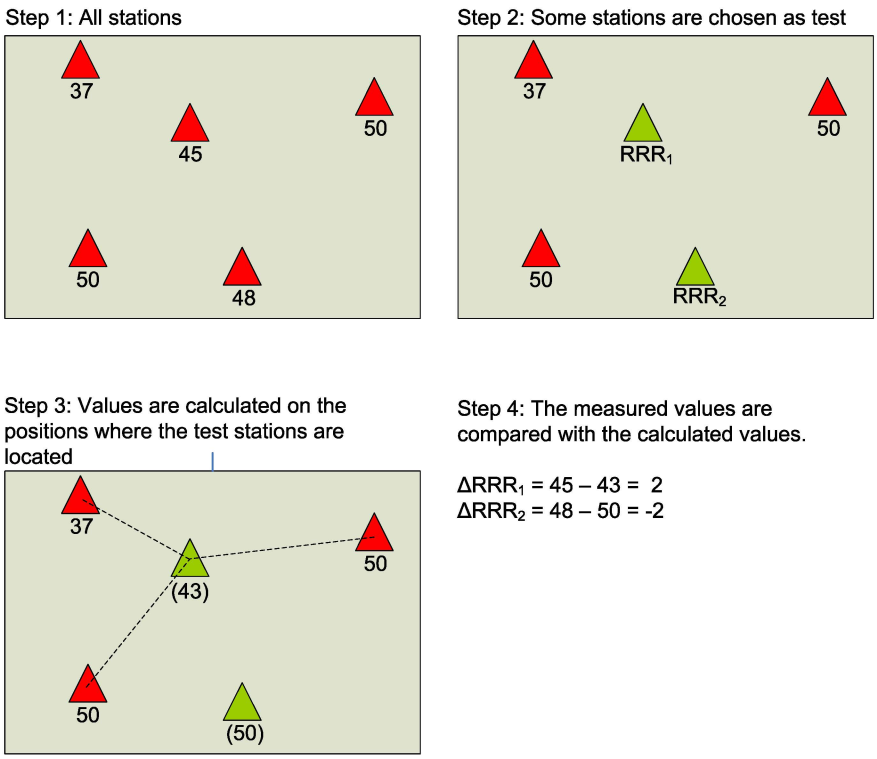

Figure 8. Procedures in step 1-4 for using full-cross validations on a set of data.

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Validation is performed for mean temperature (for year 2002). Histogram for validation of temperature is shown in Figure 10.

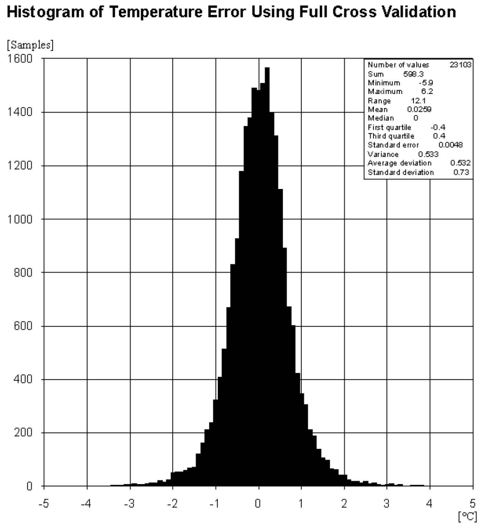


Figure 9. Histogram for validation temperature.

The observation network is embedded in the dataset, hence the interpolation algorithm is calculated so that the grid cell that geographically contains a station, always will have the observed value. Thus, if you inspect the grid network, the original dataset will appear from this.
Interpolated values will never exceed the values of the original data the grid network was calculated from. This ensures that “false” records will never appear from the interpolation.

For a detailed description of the interpolation, see Wang & Scharling (2010).

7. EVA Methods and Results

7.1 Generalized Extreme Value

We wish to consider the likely return levels of temperatures in the dataset, and there are several ways of doing this. The methods have in common that a statement is made, based on an assumed model for the distribution of extremes, about the size of an event (the return level) which occurs at least once in a given period (the return period). As we presently have data for 20 years, we could choose to mainly empirically discuss short return periods (e.g. 2, 5 and 10 years) on the basis of the observed data, but fitting an appropriate model to the extremes of the observed data gives us a tool to say something about events that occur beyond the length of the data period itself. There is a limit, of course, and for very long return periods the theoretical return levels will have very large uncertainties, if the data period they are calculated from is short.

Several types of models can be chosen to perform extreme-value statistics - some are based on analysis of the collected once-a-year maxima (or minima), others are based on analysis of the events that exceed a chosen level. Some methods have a theoretical formulation that requires determination of more model-parameters than other methods do. In general, one should be parsimonious with the choice of models given a fixed set of data. The more parameters you have to determine from the same data the less precise each may be. But, of overarching importance is the choice of a method that is sufficient given expectations from the type of data at hand. Here, we are dealing with temperatures, and choosing either to fit Generalized Extreme Value (GEV) distributions to annual extremes, or fitting Generalized Pareto distributions to data exceeding a level are quite standard (See e.g Coles, 2001). Other types of data, such as winds, precipitation, tides, etc., may be better fit using other models.

We shall fit a GEV model to extremes of temperature data (highest or lowest each year, subdivided on a monthly basis).

The GEV model contains three model parameters -μ, σ and ξ, called 'location', 'scale' and 'shape'. With these three parameters known for a given fit to data, we can use the following formula (p. 49 in Coles, 2001) and the fitted parameters to generate theoretical return levels (z_p) for given return periods (1=p):

$$z_p = \begin{cases} \mu - \frac{\sigma}{\xi}\left[1 - y_p^{-\xi}\right], if \xi \neq 0 \\ \mu - \sigma log y_p, if \xi = 0 \end{cases} \tag{3}$$

where y_p = -log(1-*p*), (here, log is the natural log function), and ***p*** is the probability of the event - e.g. *p* = 0.01 in annual data implies a hundred-year event. When z_p is plotted against y_p the plot is called a **return-level** plot. The parameter ξ, determines if the distribution is upward bounded or not - negative values of ξ correspond to bounded distributions. ξ is typically the least well-determined of the three GEV parameters.

If the shape parameter ξ is negative and well determined it is possible to also calculate the expected upper limit at infinite return period defined by:

$$z_\infty = \mu - \frac{\sigma}{\xi} \text{ (p. 56 Coles, 2001).} \tag{4}$$

The extreme-value analysis performed here assumes that the data are stationary. We apply a statistical test (the two-sided Kolmogorov-Smirnov test (e.g see Arnold and Emerson, 2011)) to see if this is the case – or rather, we test if the first half of the data are from the same distribution as the second half which is not true if there is a trend in level or variance between the first and second halves of the data.

Table 4 shows that this may not be the case - for instance, in month 7 first vs. second halves of neither T_{min}, T_{mean} nor T_{max} can be said to be drawn from the same distribution. This is either due to different means or different distribution shapes (e.g. due to different variance), or both. Only months 4, 9 and 11 seem to pass the KS2 test. In principle, we should therefore not go ahead with calculating return levels from these data. At the moment, however, we shall assume stationarity and suggest that we follow up later with remedies for the possible problem.

One remedy can be to model the extreme-value distributions with co-variates.

Suggestions for co-variates include time, as well as the North Atlantic Oscillation index which is a measure of the interaction between surface pressure over the North Atlantic and the path of low-pressure systems – low-pressure systems passing over Denmark influence temperature, winds and precipitation. Another remedy could be to fit stationary EV-models to subsections of the data and extrapolate the fits.

Table 4. Two-sided Kolmogorov-Smirnov test p-values on first and second half of each month's data (columns), for T_{min}, T_{mean} and T_{max}. In **boldface** are shown those *p*-values that are above the critical *p*-value (here $p_{crit} = 0.05$) and thus indicate similarity of the two distributions.

Series	1	2	3	4	5	6	7	8	9	10	11	12
T_{min}	0.029	0.003	0.003	**0.369**	**0.166**	**0.440**	0.000	0.000	**0.825**	0.008	**0.860**	0.000
T_{mean}	**0.056**	0.001	0.006	**0.462**	0.049	0.039	0.009	0.045	**0.077**	0.001	**0.931**	0.000
T_{max}	**0.286**	**0.166**	**0.052**	**0.665**	**0.114**	0.012	0.010	**0.514**	**0.772**	0.008	**0.294**	0.000

We now show two examples of the application of GEV-theory to the chosen data set.

7.2 Example 1: T_{mean} 2006-2020 Denmark

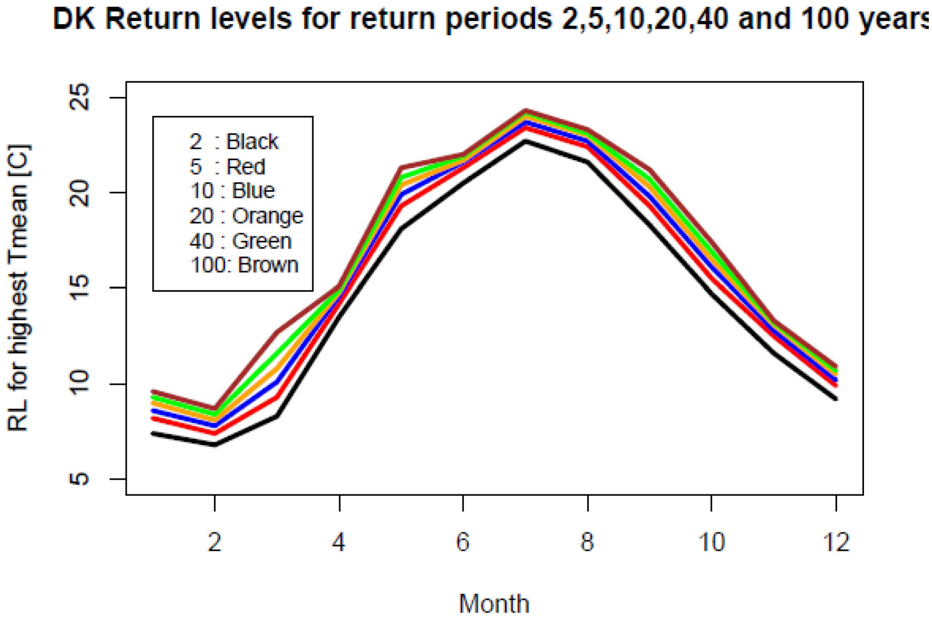
The previously described daily temperature values are now used for analysis. For this example, mean temperature is used only. We calculate return levels for a set of return periods (2, 5, 10, 20, 40 and 100 years) on a monthly basis.

Figure 10 shows the return levels for each month calculated from the observed data, 2006-2020. Figure 11 shows also the 5 and 95%iles of these curves, based on Monte Carlo resampling (with replacement) of each month's data which gives a broad view of the sampling uncertainties to be expected on the curves in Figure 10.

On the last panel of Figure 11 (for 100 year RP) we see that the 2-year and 100-year RP return levels are significantly different. Overall we note how similar the return levels for different return periods are. This is likely because we are here looking at daily T_{mean} values across Denmark.
The GEV parameter "shape" is generally negative, for both T_{mean} maxima and minima (not shown), suggesting that the distribution of these extremes are bounded (from above for maxima, from below for minima) – see Section 7.1.

The broader uncertainty intervals in the cold months are noted.

The GEV procedure used (*fevd()* from the ***R*** library *extRemes*) does not always converge under resampling and in those cases the results have been omitted.



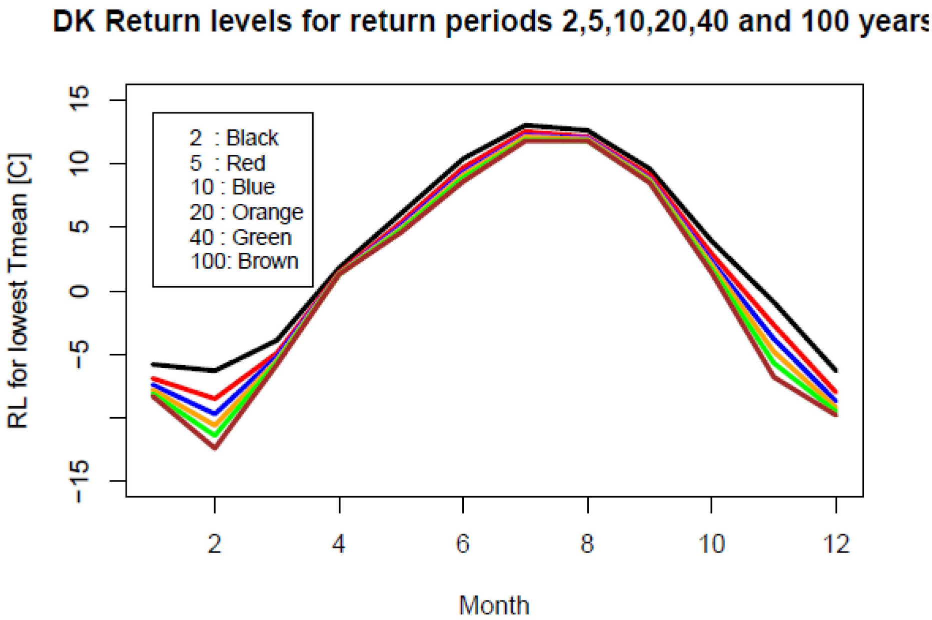


Figure 10. 2, 5, 10, 20, 40 and 100 years return levels for the monthly 'highest extremes' in T_{mean} (upper panel) and monthly 'lowest extremes' in T_{mean} (lower panel). Data from 2006-2020.

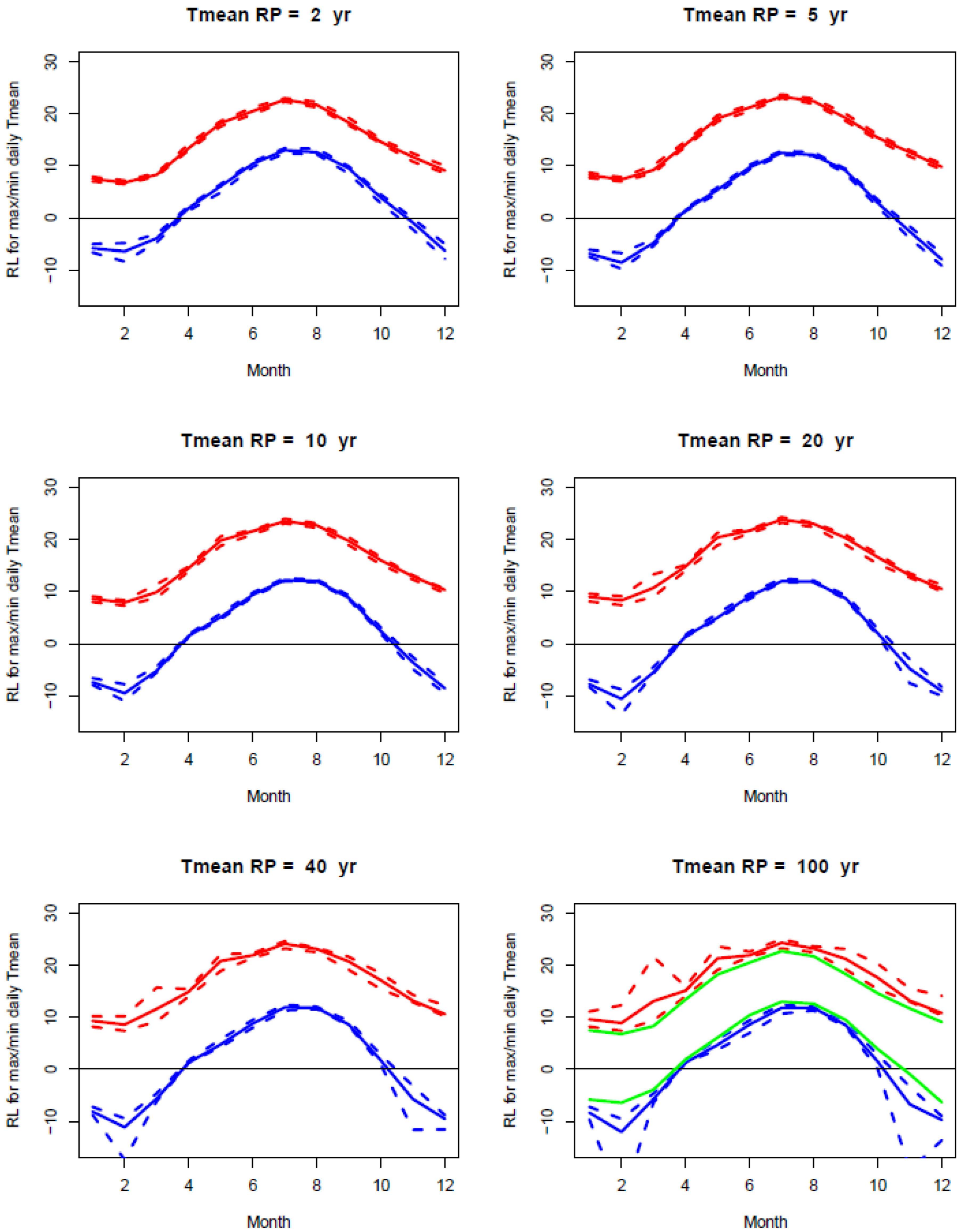


Figure 11. 2, 5, 10, 20, 40 and 100 year return levels, with 5, 50 and 95%iles shown for monthly high T_{mean} (red curves) and low T_{mean} (blue curves) for data from 2006-2020. The green curve in the last panel is the 2-year RL 50%ile. The dotted lines in the last panel show the actual observed max and min values of T_{mean}. A three-parameter GEV model was fitted to the extremes.

7.3 Example 2 T_{min}, T_{mean} and T_{max} 2001-2020 Denmark

We now show results for a longer period of data (2001-2020). Again it is the daily values for Denmark, but this time we look at T_{min}, T_{mean} and T_{max}. Results are shown in Figures 12-14.
We note first that the results for T_{mean} are very similar to those in Figure 11 where less data was used (2006-2020).

The 50%iles are thus robust under data volume. The uncertainty bands, however, have improved under addition of data - especially those for the 100-year return level.

We found that variability in calculated extreme levels, and in particular their upper and lower significance-bands were sensitive to outliers in the data.

As a consequence, we set the shape parameter ξ to 0 and thus fitted for just the location and scale parameters μ and σ. This choice is not without potential consequences - we have lowered the sensitivity to outliers but may have increased the bias on parameter estimates. For this demonstration project, and in view of inspection of the extreme return levels and their validation, we find it appropriate to proceed in this way, but in the future, if larger data-amounts became available, we would re-evaluate this choice.

7.4 Summary

We have inspected Danish values for minimum, mean and maximum temperatures in the dataset from 2001-2020. We have calculated return levels for the highest and lowest of the three observables T_{min}, T_{mean} and T_{max}.

We have tested robustness by extending an initial data sample from 2006-2020 to data from 2001-2020 and noted that mainly the confidence bands of the (40 and) 100-year return levels responded to this by becoming (slightly) less erratic. This indicates that adding further amounts of data, with time, would stabilize also the high return level confidence limits.

We have tested if the input data passes a test for stationarity, and found that this may not be the case. We suggest a follow-up study of this, and suggest using extreme-value modeling based on co-variate modelling as a remedy.

In Example 1 we used GEV-distributions to model extreme values and allowed a shape parameter in the model. On physical grounds we know that temperatures in the climate system are always limited upwards and downwards. Additionally, an unfortunate sensitivity in the confidence bands to the presence of data outliers made is favorable to fit GEV models with the shape parameter ξ set to zero.

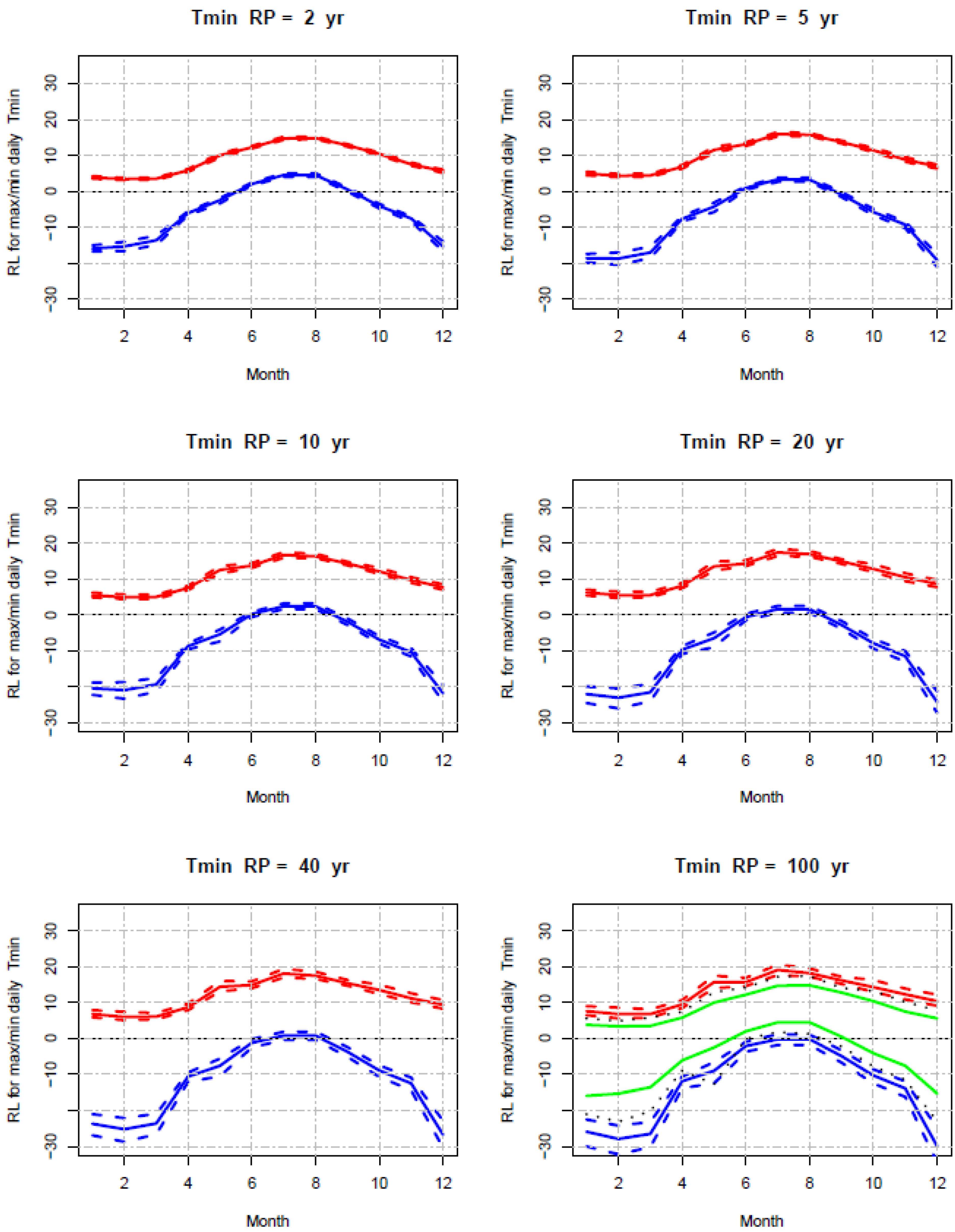


Figure 12. 2, 5, 10, 20, 40 and 100 year return levels, with 5, 50 and 95%iles shown for monthly high T_{min} (red curves) and low T_{min} (blue curves) for data from 2001-2020. The green curve in the last panel is the 2-year RL 50%ile. The dotted lines in the last panel show the actual observed max and min values of T_{min}. A two-parameter GEV model was fitted to the extremes here, which has contributed to the stabilization of the confidence bands –same in figure 13 and 14.

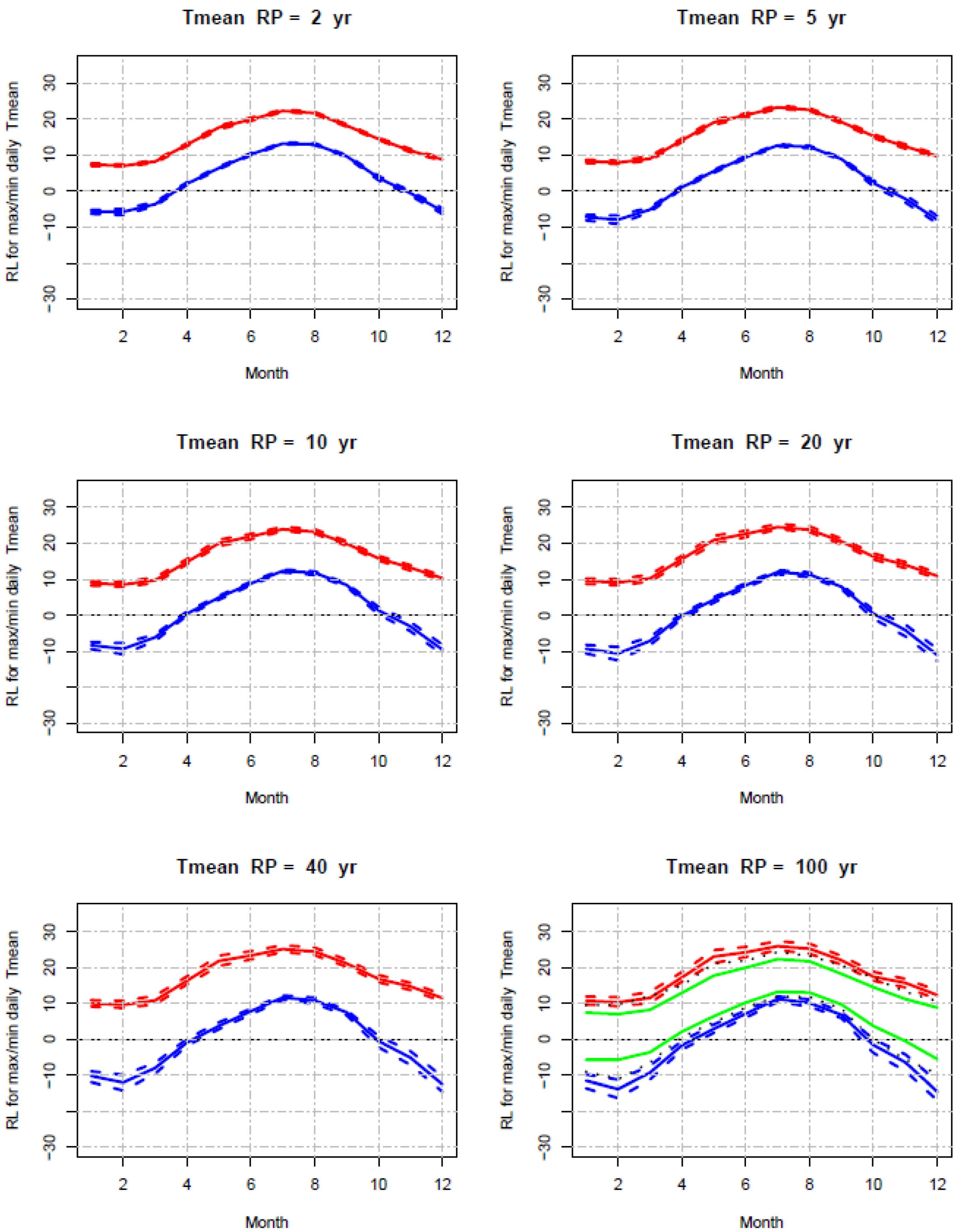


Figure 13. 2, 5, 10, 20, 40 and 100 year return levels, with 5, 50 and 95%iles shown for monthly high T_{mean} (red curves) and low T_{mean} (blue curves) for data from 2001-2020. The green curve in the last panel is the 2-year RL 50%ile. The dotted lines in the last panel show the actual observed max and min values of T_{mean}. A two-parameter GEV model was fitted to the extremes here, which has contributed to the stabilization of the confidence bands.

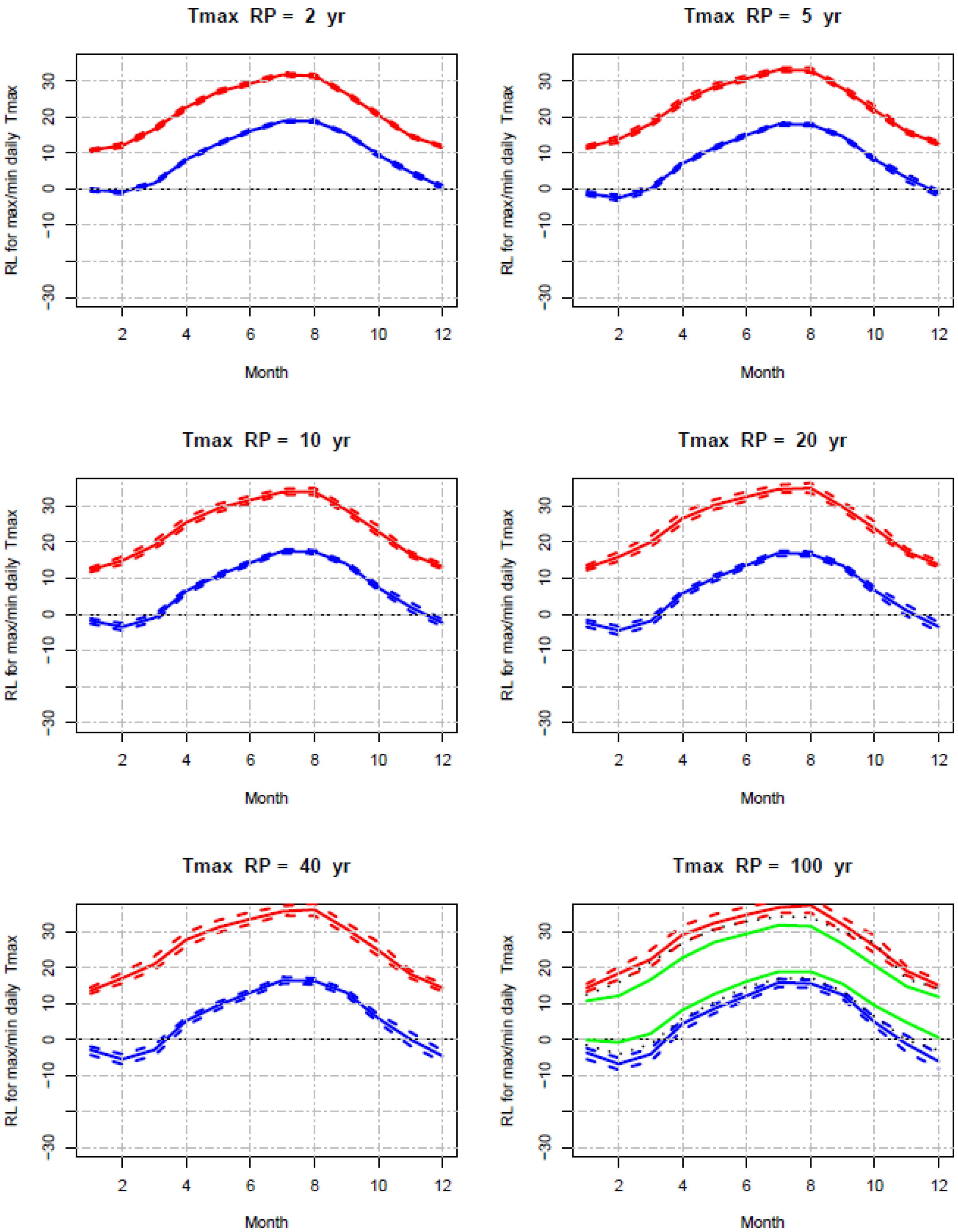


Figure 14. 2, 5, 10, 20, 40 and 100 year return levels, with 5, 50 and 95%iles shown for monthly high T_{max} (red curves) and low T_{maxn} (blue curves) for data from 2001-2020. The green curve in the last panel is the 2-year RL 50%ile. The dotted lines in the last panel show the actual observed max and min values of T_{max}. A two-parameter GEV model was fitted to the extremes here, which has contributed to the stabilization of the confidence bands.

Table 5. EVA results for *daily minimum temperature* (°C): Minimum extreme return levels (50% confidence level) and uncertainty levels (5%-95% confidence intervals)

	2 yr RP			5 yr RP			10 yr RP			20 yr RP			40 yr RP			100 YR RP		
Month	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%
Jan	-16.7	**-15.9**	-15	-19.8	**-18.6**	-17.4	-22.2	**-20.4**	-18.8	-24.5	**-22**	-19.9	-26.9	**-23.7**	-21	-30	**-25.9**	-22.5
Feb	-16.6	**-15.3**	-14	-20.4	**-18.7**	-17	-23.3	**-20.9**	-18.7	-26	**-23**	-20.4	-28.7	**-25.2**	-22.1	-32.2	**-27.9**	-24.2
Mar	-14.9	**-13.6**	-12.3	-18.6	**-17**	-15.4	-21.3	**-19.3**	-17.5	-24	**-21.5**	-19.2	-26.6	**-23.6**	-20.9	-30.2	**-26.5**	-23.2
Apr	-6.7	**-6.1**	-5.6	-8.4	**-7.7**	-7	-9.6	**-8.7**	-7.9	-10.9	**-9.7**	-8.7	-12.1	**-10.6**	-9.5	-13.8	**-11.9**	-10.6
Maj	-3.4	**-2.5**	-1.9	-5.8	**-4.3**	-3.2	-7.4	**-5.4**	-4.1	-9	**-6.5**	-4.9	-10.5	**-7.6**	-5.6	-12.5	**-9**	-6.6
Jun	1.6	**2**	2.4	0.4	**0.9**	1.4	-0.6	**0.2**	0.8	-1.5	**-0.6**	0.3	-2.5	**-1.2**	-0.2	-3.7	**-2.1**	-0.9
Jul	4	**4.5**	5	2.6	**3.3**	3.8	1.6	**2.4**	3.2	0.6	**1.6**	2.5	-0.4	**0.8**	1.9	-1.8	**-0.2**	1.2
Aug	4	**4.5**	5	2.6	**3.2**	3.8	1.6	**2.4**	3.2	0.6	**1.6**	2.5	-0.4	**0.8**	1.9	-1.8	**-0.2**	1.1
Sep	-0.1	**0.5**	1	-1.6	**-0.9**	-0.2	-2.8	**-1.9**	-1	-3.9	**-2.8**	-1.7	-5.1	**-3.7**	-2.4	-6.7	**-4.9**	-3.3
Okt	-4.8	**-4.1**	-3.5	-6.7	**-5.8**	-4.9	-8	**-6.9**	-5.9	-9.4	**-7.9**	-6.7	-10.7	**-8.9**	-7.5	-12.5	**-10.3**	-8.6
Nov	-8.4	**-7.6**	-7	-10.2	**-9.3**	-8.5	-11.6	**-10.4**	-9.3	-13	**-11.5**	-10.1	-14.4	**-12.5**	-10.9	-16.2	**-13.9**	-11.9
Dec	-16.6	**-15.3**	-13.9	-21	**-19.1**	-17.4	-24.1	**-21.7**	-19.4	-27.2	**-24.2**	-21.2	-30.3	**-26.7**	-22.8	-34.4	**-29.9**	-24.8

Table 6. EVA results for *daily minimum temperature* (°C): Maximum extreme return levels (50% confidence level) and uncertainty levels (5%-95% confidence intervals)

	2 yr RP			5 yr RP			10 yr RP			20 yr RP			40 yr RP			100 YR RP		
Month	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%
Jan	3.5	**3.8**	4.2	4.4	**4.9**	5.4	4.9	**5.5**	6.2	5.4	**6.2**	7	5.9	**6.8**	7.9	6.5	**7.6**	9
Feb	3.1	**3.4**	3.7	3.9	**4.3**	4.8	4.4	**4.9**	5.7	4.7	**5.5**	6.5	5.1	**6**	7.4	5.6	**6.8**	8.6
Mar	3.2	**3.5**	3.8	4	**4.4**	4.9	4.4	**5**	5.6	4.9	**5.5**	6.4	5.3	**6.1**	7.1	5.8	**6.8**	8.1
Apr	5.4	**5.8**	6.2	6.3	**6.8**	7.3	6.9	**7.5**	8.1	7.4	**8.1**	8.9	7.9	**8.7**	9.7	8.5	**9.5**	10.7
Maj	9.4	**10**	10.6	10.7	**11.5**	12.4	11.5	**12.5**	13.6	12.3	**13.5**	14.8	13.1	**14.4**	16	14	**15.7**	17.5
Jun	11.9	**12.2**	12.6	12.6	**13.1**	13.6	13.1	**13.7**	14.4	13.6	**14.3**	15.2	14	**14.9**	15.9	14.6	**15.7**	16.9
Jul	14.3	**14.7**	15.2	15.3	**15.9**	16.5	15.9	**16.7**	17.5	16.4	**17.4**	18.4	17	**18.1**	19.4	17.7	**19.1**	20.6
Aug	14.5	**14.8**	15.2	15.3	**15.7**	16.2	15.8	**16.3**	17	16.2	**16.9**	17.8	16.6	**17.5**	18.6	17.2	**18.2**	19.5
Sep	12.4	**12.8**	13.1	13.3	**13.7**	14.1	13.8	**14.3**	14.8	14.2	**14.9**	15.5	14.7	**15.4**	16.3	15.2	**16.2**	17.2
Okt	10	**10.4**	10.8	10.9	**11.4**	12.1	11.4	**12.1**	13.1	11.9	**12.8**	14.1	12.4	**13.5**	15	13	**14.3**	16.3
Nov	7.1	**7.5**	8	8.2	**8.8**	9.5	8.9	**9.7**	10.5	9.4	**10.5**	11.6	10.1	**11.2**	12.6	10.9	**12.3**	13.9
Dec	5.1	**5.6**	6	6.3	**6.9**	7.4	7	**7.7**	8.5	7.7	**8.6**	9.7	8.3	**9.4**	10.7	9.1	**10.4**	12.2

Table 7. EVA results for *daily mean temperature* (°C): Minimum extreme return levels (50% confidence level) and uncertainty levels (5%-95% confidence intervals)

	2 yr RP			5 yr RP			10 yr RP			20 yr RP			40 yr RP			100 YR RP		
Month	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%
Jan	-6.3	**-5.7**	-5.2	-8.1	**-7.2**	-6.6	-9.4	**-8.3**	-7.4	-10.6	**-9.3**	-8.2	-11.9	**-10.2**	-8.9	-13.6	**-11.5**	-9.8
Feb	-6.6	**-5.7**	-4.9	-9.1	**-7.9**	-6.7	-10.8	**-9.3**	-7.7	-12.5	**-10.6**	-8.8	-14.2	**-12**	-9.9	-16.3	**-13.8**	-11.2
Mar	-4.1	**-3.6**	-3	-5.8	**-5.1**	-4.5	-7.1	**-6.1**	-5.3	-8.3	**-7.1**	-6	-9.6	**-8**	-6.6	-11.2	**-9.3**	-7.4
Apr	1.8	**2.1**	2.5	0.7	**1.1**	1.5	-0.1	**0.4**	1	-1	**-0.2**	0.5	-1.8	**-0.9**	0.1	-2.9	**-1.7**	-0.5
Maj	6.1	**6.4**	6.8	5.1	**5.5**	5.9	4.3	**4.9**	5.4	3.5	**4.3**	5	2.8	**3.7**	4.7	1.8	**2.9**	4.1
Jun	9.9	**10.2**	10.5	9	**9.4**	9.7	8.3	**8.8**	9.3	7.6	**8.3**	8.9	6.9	**7.8**	8.5	6	**7.1**	8
Jul	13	**13.2**	13.5	12.4	**12.7**	13	11.9	**12.3**	12.7	11.4	**12**	12.5	11	**11.6**	12.2	10.4	**11.2**	11.9
Aug	12.8	**13**	13.3	11.9	**12.3**	12.6	11.2	**11.8**	12.2	10.6	**11.3**	11.9	9.9	**10.8**	11.6	9.1	**10.1**	11.1
Sep	9.4	**9.7**	10	8.6	**8.9**	9.2	7.9	**8.4**	8.8	7.3	**7.9**	8.4	6.7	**7.5**	8.1	5.9	**6.8**	7.6
Okt	3.1	**3.7**	4.2	1.4	**2.3**	3	0.2	**1.3**	2.3	-1	**0.4**	1.5	-2.2	**-0.5**	0.9	-3.8	**-1.6**	0
Nov	-1.2	**-0.5**	0.1	-3.1	**-2.1**	-1.1	-4.4	**-3.1**	-1.9	-5.8	**-4.1**	-2.6	-7.1	**-5.1**	-3.3	-8.8	**-6.4**	-4.2
Dec	-6.3	**-5.5**	-4.7	-8.9	**-7.9**	-6.8	-10.8	**-9.5**	-8.1	-12.7	**-11**	-9.3	-14.5	**-12.5**	-10.4	-16.9	**-14.5**	-11.7

Table 8. EVA results for *daily mean temperature* (°C): Maximum extreme return levels (50% confidence level) and uncertainty levels (5%-95% confidence intervals)

	2 yr RP			5 yr RP			10 yr RP			20 yr RP			40 yr RP			100 YR RP		
Month	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%
Jan	7	**7.4**	7.7	7.8	**8.3**	8.7	8.2	**8.9**	9.5	8.7	**9.5**	10.2	9.1	**10**	10.9	9.7	**10.7**	11.9
Feb	6.8	**7**	7.4	7.5	**7.9**	8.4	7.9	**8.5**	9.2	8.3	**9.1**	10	8.7	**9.6**	10.7	9.2	**10.3**	11.7
Mar	7.8	**8.2**	8.5	8.5	**9**	9.7	8.9	**9.7**	10.5	9.3	**10.2**	11.3	9.7	**10.8**	12.1	10.1	**11.5**	13.2
Apr	12.5	**12.9**	13.3	13.6	**14.1**	14.6	14.2	**14.8**	15.6	14.7	**15.6**	16.5	15.3	**16.3**	17.5	16	**17.2**	18.7
Maj	17.1	**17.7**	18.2	18.4	**19.1**	19.8	19.2	**20**	21	19.8	**20.9**	22.1	20.4	**21.8**	23.3	21.2	**23**	24.8
Jun	19.5	**19.9**	20.4	20.5	**21.1**	21.6	21.1	**21.9**	22.6	21.7	**22.6**	23.5	22.2	**23.3**	24.5	22.9	**24.2**	25.7
Jul	22	**22.3**	22.7	22.9	**23.3**	23.8	23.4	**23.9**	24.6	23.9	**24.5**	25.4	24.3	**25.1**	26.2	24.8	**25.9**	27.3
Aug	21.4	**21.7**	22	22.3	**22.6**	23	22.7	**23.2**	23.9	23.1	**23.8**	24.7	23.4	**24.4**	25.5	23.8	**25.2**	26.6
Sep	17.9	**18.2**	18.6	18.8	**19.2**	19.8	19.3	**19.9**	20.6	19.7	**20.5**	21.3	20.2	**21.1**	22.1	20.8	**22**	23.2
Okt	14.2	**14.5**	14.8	14.8	**15.3**	15.8	15.2	**15.8**	16.5	15.5	**16.3**	17.2	15.9	**16.8**	17.9	16.3	**17.4**	18.8
Nov	10.8	**11.2**	11.7	11.8	**12.4**	13	12.4	**13.2**	13.9	13.1	**14**	14.8	13.7	**14.7**	15.7	14.4	**15.7**	16.8
Dec	8.4	**8.8**	9.1	9.3	**9.7**	10.2	9.8	**10.3**	11	10.2	**10.9**	11.8	10.6	**11.5**	12.5	11.2	**12.3**	13.6

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Table 9. EVA results for *daily maximum temperature* (°C): Minimum extreme return levels (50% confidence level) and uncertainty levels (5%-95% confidence intervals)

	2 yr RP			5 yr RP			10 yr RP			20 yr RP			40 yr RP			100 YR RP		
Month	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%
Jan	-0.6	**-0.2**	0.1	-1.7	**-1.2**	-0.7	-2.6	**-1.8**	-1.2	-3.5	**-2.4**	-1.6	-4.3	**-2.9**	-2	-5.5	**-3.7**	-2.5
Feb	-1.4	**-0.8**	-0.3	-3.2	**-2.4**	-1.6	-4.5	**-3.5**	-2.5	-5.7	**-4.5**	-3.3	-6.9	**-5.5**	-4.1	-8.4	**-6.8**	-5.1
Mar	1.1	**1.6**	2.1	-0.7	**0.1**	0.7	-2	**-1**	-0.1	-3.3	**-1.9**	-0.9	-4.5	**-2.9**	-1.5	-6.1	**-4.1**	-2.5
Apr	7.8	**8.2**	8.5	6.6	**7.1**	7.6	5.7	**6.5**	7.1	4.8	**5.8**	6.6	3.9	**5.2**	6.1	2.7	**4.4**	5.5
Maj	12.2	**12.6**	13	11	**11.5**	12	10.1	**10.8**	11.4	9.2	**10.1**	10.9	8.4	**9.5**	10.4	7.3	**8.6**	9.8
Jun	15.8	**16.1**	16.6	14.6	**15**	15.5	13.7	**14.3**	14.9	12.9	**13.6**	14.3	12	**13**	13.8	10.9	**12.1**	13.2
Jul	18.5	**18.8**	19.1	17.6	**18**	18.4	16.9	**17.5**	18	16.2	**17**	17.6	15.5	**16.4**	17.3	14.6	**15.8**	16.8
Aug	18.5	**18.8**	19.2	17.5	**17.9**	18.3	16.8	**17.3**	17.8	16	**16.7**	17.4	15.3	**16.2**	17	14.3	**15.5**	16.5
Sep	15.1	**15.4**	15.7	14.1	**14.6**	15	13.5	**14**	14.6	12.8	**13.5**	14.2	12.1	**13**	13.9	11.2	**12.4**	13.4
Okt	8.9	**9.4**	9.8	7.6	**8.2**	8.7	6.6	**7.3**	8.1	5.6	**6.6**	7.5	4.7	**5.8**	6.9	3.5	**4.8**	6.1
Nov	4	**4.7**	5.3	2.1	**3.1**	4.1	0.8	**2**	3.3	-0.5	**1**	2.6	-1.8	**0**	2	-3.5	**-1.3**	1
Dec	-0.1	**0.5**	1.1	-2	**-1.2**	-0.5	-3.4	**-2.4**	-1.5	-4.8	**-3.5**	-2.3	-6.2	**-4.6**	-3.1	-8.1	**-6.1**	-4.1

Table 10. EVA results for *daily maximum temperature* (°C): Maximum extreme return levels (50% confidence level) and uncertainty levels (5%-95% confidence intervals)

	2 yr RP			5 yr RP			10 yr RP			20 yr RP			40 yr RP			100 YR RP		
Month	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%	5%	**50%**	95%
Jan	10.3	**10.7**	11	11.2	**11.6**	12.1	11.7	**12.3**	12.9	12.2	**12.9**	13.6	12.7	**13.5**	14.4	13.3	**14.3**	15.4
Feb	11.5	**12.1**	12.8	12.9	**13.8**	14.7	13.8	**14.9**	16	14.7	**15.9**	17.3	15.6	**17**	18.5	16.6	**18.3**	20.1
Mar	16	**16.6**	17.2	17.4	**18.1**	19	18.1	**19.1**	20.4	18.7	**20**	21.7	19.4	**20.9**	23.1	20.2	**22.2**	24.8
Apr	22	**22.7**	23.4	23.5	**24.4**	25.4	24.4	**25.5**	26.9	25.2	**26.6**	28.3	25.9	**27.7**	29.7	27	**29.1**	31.6
Maj	26.5	**27**	27.6	27.7	**28.4**	29.3	28.4	**29.4**	30.6	29.1	**30.3**	31.8	29.7	**31.1**	33.1	30.5	**32.3**	34.7
Jun	28.9	**29.3**	29.9	30.1	**30.7**	31.5	30.8	**31.7**	32.8	31.4	**32.6**	34	32	**33.4**	35.3	32.9	**34.6**	36.8
Jul	31.3	**31.7**	32.3	32.5	**33.1**	33.7	33.2	**33.9**	34.8	33.8	**34.7**	35.9	34.4	**35.5**	37	35.2	**36.6**	38.4
Aug	30.9	**31.4**	31.9	32.4	**32.9**	33.7	33.1	**34**	35	33.7	**35**	36.4	34.3	**36**	37.7	35.1	**37.2**	39.4
Sep	26	**26.5**	27	27.3	**27.9**	28.6	28	**28.9**	29.9	28.7	**29.8**	31.1	29.3	**30.7**	32.3	30.2	**31.9**	33.8
Okt	20	**20.5**	21.2	21.1	**21.9**	23.1	21.8	**22.9**	24.5	22.5	**23.8**	25.8	23.1	**24.7**	27	23.9	**25.9**	28.8
Nov	14.3	**14.7**	15.1	15.3	**15.9**	16.4	15.9	**16.6**	17.3	16.5	**17.3**	18.2	17	**18.1**	19.1	17.8	**19**	20.2
Dec	11.4	**11.8**	12.1	12.1	**12.6**	13.2	12.5	**13.2**	14	13	**13.8**	14.8	13.3	**14.3**	15.5	13.8	**15**	16.5

7.5 Examples of return periods

Having stated the underlying assumptions and conditions of the performed analysis we try to give some examples of one way to use the calculated return levels.

7.5.1 Minimum temperature

As an example, it is a 1 in 40 year incident that the temperature in Denmark gets down to -23.7°C, or lower, in January. Thus, the return period for reaching this temperature is 40 years. On the other hand, the return period is only 2 years for a minimum temperature of -15.9 °C on a day in January.
When inspecting the summer months, it is a 1 in 40-year incident that the minimum temperature in Denmark is 0.8°C, or lower, in August, though it is only a 1 in 2-year incident that the minimum temperature is 4.5 °C or higher.

7.5.2 Mean temperature

As an example, it is a 1 in 40 year incident, that the daily mean temperature in Denmark reaches 10.0 °C, or higher, in January. Thus, the return period for passing this temperature is 40 years. On the other hand, the return period is only 2 years for a daily mean temperature of 7.4 °C in January.
When inspecting the summer months, it is a 1 in 40-year incident that the daily mean temperature in Denmark reaches 24.4 °C, or higher, in August, though it is only a 1 in 2-year incident that the daily mean temperature reaches 21.7 °C or higher.

7.5.3 Maximum temperature

As an example, it is a 1 in 40 year incident, that the daily maximum temperature in Denmark reaches 13.5 °C, or higher, in January. Thus, the return period for passing this temperature is 40 years. On the other hand, the return period is only 2 years for a maximum temperature of 10.7 °C or higher in January.
When inspecting the summer months, it is a 1 in 40-year incident that the daily maximum temperature in Denmark reaches 36.0 °C, or higher, in August, though it is only a 1 in 2-year incident that the daily mean temperature reaches 31.4 °C or higher.

8. Discussion

The EVA applied for this study assumes that the input data are stationary as described in section 7. The statistical test showed that data did not meet this requirement, however a visual inspection of data reveal that the input data are not obviously showing trends in the mean or changes in variance with time. Therefore return levels were still calculated and it is suggested that these potential problems are handled later on by extending the modelling paradigm to the non-stationary domain with co-variates modelling the changing features of the data.

When interpreting the calculated return levels of an EVA one must at all times consider, that the type of analysis is an estimate of the probability of extreme events likely to occur based on an observed data set.

It must be understood by the non-statistician that statements about return levels for long return periods are not predictions of future events. The calculations merely say – given these data, we understand that in a much larger body of *identically distributed data* we could expect to see such and such extreme events, so far unobserved in our smaller sample.

As earlier mentioned we intended to make up for the lack of stationarity with remedies, e.g. adding co-variates to improve the statistical model or to fit stationary EV-models to subsections of the data and extrapolate the fits.

With this project it was intended to extract knowledge of the extreme events having occurred within the data set – or even extreme events outside the observed data range.

We attempt to describe the possibility of those extreme events that are possible to quantify, though, exclusively for a situation, where the data continuously looks like the data used in the analysis.

As shown in this work, we can see indications that the short data-segment we have available is non-stationary. Since our report is exploratory in nature we chose to proceed with the stationarity-based analysis anyway. In a potential extension of this project we would work to gather more archival data, thus extending the time-coverage which would help us better test and understand the non-stationarity present. We also call for the use of statistical methods that can explicitly handle non-stationarity, which would be a reasonable step to take if better data-coverage was present.

9. References

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